TECHNICAL NOTE: LOWER TOLERANCE LIMIT APPROACH TO EQUATION-BASED RATIONAL DESIGN VALUES FOR T-SHAPED MORTISE AND TENON JOINTS

*C. A. Eckelman**

Professor
E-mail: eckelmac@purdue.edu

E. Haviarova†*

Associate Professor
Department of Forestry and Natural Resources
Purdue University, West Lafayette, IN
E-mail: ehaviar@purdue.edu

A. Kasal

Professor
E-mail: alikasal@mu.edu.tr

Y. Z. Erdil

Professor
Department of Wood Science and Industrial Engineering
Mugla Sitki Kocman University
48000, Kotekli/Mugla, Turkey
E-mail: erdil@mu.edu.tr

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Abstract. A nonlinear regression expression was fitted to the test data obtained from a study of the bending moment capacity of 320 rectangular T-shaped mortise and tenon furniture joints consisting of 64 configurations of five specimens each. A statistical lower tolerance limit approach was then used to explore the degree to which these values should be reduced when used for design purposes and the confidence that a user might have in these reductions. The procedure followed was to apply statistical lower tolerance limit techniques to the ratios obtained by dividing each test value by its corresponding estimated value. To gain insight into the relationship of a specific confidence–proportion level and its corresponding reduction factor on the percentage of an estimated value that could be used for design purposes, lower tolerance limits were computed for four confidence–proportion levels. The results illustrate a statistical technique that can be used to determine reduction factors and the impact of the selection of any of the given confidence–proportion levels on design values.

Keywords: Statistical lower tolerance limits, rectangular mortise and tenon joints.

INTRODUCTION

In studies of the bending moment capacities of furniture joints, researchers (Kasal et al 2008) have modeled the resulting test data by means of equations obtained from nonlinear regression analyses of the data. Such equations are useful in that they provide a means of concisely compressing the results of a study into an easily useable form that is suitable for use in design manuals, etc. A limiting factor in the use of such equations, however, is that the estimates are essentially only "averages" of the test values

* Corresponding author
† SWST member

obtained for any joint configuration. Hence, for design purposes, the values estimated by these equations should be reduced to take into account the fact that about half of the test values on which the expression is based were less than the corresponding estimated value.

Statistical lower tolerance limit (LTL) techniques provide a means of obtaining rational insights into the consequences of choosing specific reductions in estimated values. In the case of test data for a single specific joint construction, eg LTL techniques can be used to provide a specified degree of confidence that a specific percentage of joints of like construction could be expected to have capacities greater than some chosen value (Natrella 1963; Ostle 1963; Link 1985).

The application of statistical LTLs to the differences between estimated and test values for joints representing a variety of constructions presents a somewhat unique use of LTL techniques, and given the lack of precedents, it is useful to examine the results of such an application. In the paper which follows, the test results obtained by Kasal et al (2013) for 320 rectangular mortise and tenon joint specimens (2 species $\times$ 2 glue types $\times$ 4 lengths $\times$ 4 widths $\times$ 5 replicates) and the estimates provided by the equation fitted to them are analyzed to obtain insights into the above questions. The primary purpose of the analysis was to obtain insights into the relationship between chosen confidence–proportion levels and the LTLs obtained—as a proportion of the values estimated by the regression equation. Results of the study are given in the following sections.

OBJECTIVES

The primary objective of the study was to explore how statistical LTLs might be used to rationalize the differences between test and estimated joint capacity values for design purposes. A specific objective was to illustrate the effect of the selection of given confidence–proportion level on the reduction of estimated values.

MATERIALS AND METHODS

Description of Specimens and its Construction

The geometries of the tenons used in the study of Kasal et al (2013) are illustrated in fig 1. Half of the joints were constructed of Turkish beech and half of Scotch pine; half of the joints of each species in turn were bonded with either a 65% solids polyvinyl acetate or a polyurethane adhesive; thus, 80 specimens were constructed of Scottish pine with a polyvinyl acetate adhesive, 80 of Scottish pine with a polyurethane adhesive, 80 of European beech with a polyvinyl acetate adhesive, and 80 of European beech with a polyurethane adhesive, for a total of 320 specimens. Strength properties of the wood species are given in Table 1. Test specimens were conditioned to and tested at 12% $\pm$ 0.2% moisture content.

Method of Testing

All bending moment tests were carried out on a universal testing machine. A concentrated load was applied to the rail of a specimen at a point 300 mm from the front edge of the post so that the moment arm was 0.3 m (fig 2). Loading rate was 6 mm/min. Loading was continued until a nonrecoverable drop in load occurred.

Procedures

The procedure followed in this study was to divide each of the 320 test values obtained by Kasal et al (2013) given in Table 2 by its corresponding estimated value, where the estimated values are given by solution of the nonlinear regression equation (Kasal et al 2013)

$$M = 0.118 \times [0.25 \times (D - W) + 0.78 \times W] \times L^{0.8} \times S^{0.55} \quad (1)$$

where D refers to rail width, mm; W refers to tenon width, mm; L refers to tenon length, mm;

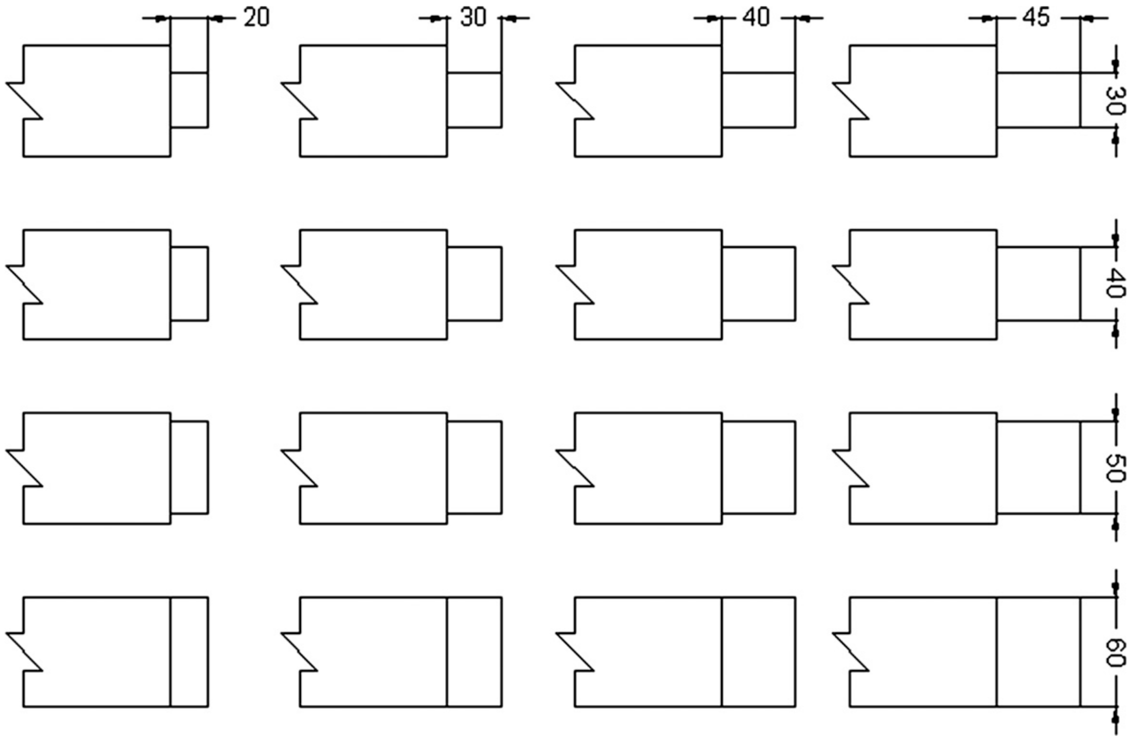

Figure 1. Cross section of all rails was 21 mm by 60 mm. Tenons were 7-mm thick with a 2-mm top and bottom edge radius.

Table 1. Strength properties and moisture content of wood species.

Wood species	MOE (N/mm^2)	Tension strength (N/mm^2)	Compression strength (N/mm^2)	Shear strength (N/mm^2)	MOR (N/mm^2)	Density (g/cm^3)	MC (%)
Turkish beech	11183	118.4	60.7	10.31	115.9	0.60	10.8
Scotch pine	10289	65.5	57.2	6.21	88.3	0.45	11.2

and *S* refers to shear strength of the wood, N/mm^2. The coefficient of determination (R^2) value of the derived expression was 0.76.

LTLs were then determined for the resulting data set (Table 3) by means of the relationship

$$\mathrm{LTL} = \bar{X} - k \times s \tag{2}$$

where LTL refers to the lower tolerance limit, $\bar{X}$ is the average of the test/estimated ratios, s is the standard deviation of the ratios, and k is the appropriate tolerance factor (Ostle 1963; Link 1985) for 320 specimens.

A "low" 75% confidence|75% proportion level and a "high" 90% confidence|90% proportion level were selected as starting points under the assumption that at the lower LTL a substantial

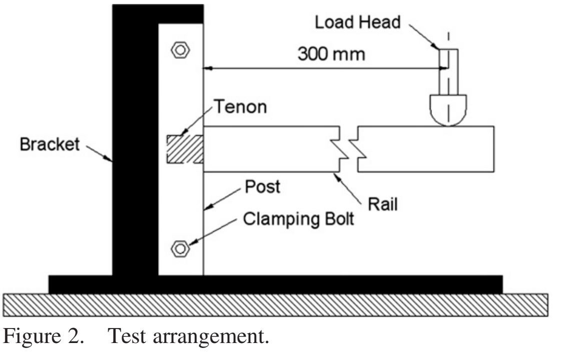

Figure 2. Test arrangement.

Table 2. Individual test results for T-shaped mortise and tenon joints.

	Tenon width (mm)															
	30				40				50				60			
	Tenon length (mm)															
	20	30	40	45	20	30	40	45	20	30	40	45	20	30	40	45
	Moment capacity (Nm)															
Beech	174	215	218	306	157	330	350	374	182	250	503	336	212	235	403	383
PVA	159	191	221	281	203	280	303	377	212	285	424	309	224	233	465	424
	130	244	212	308	180	285	297	439	217	250	477	368	191	241	494	383
	132	232	215	344	187	303	320	315	188	310	427	386	212	268	456	380
	147	214	206	341	165	288	333	330	209	297	524	386	206	274	489	462
Average	148	219	214	316	178	297	321	367	202	279	471	357	209	250	461	461
STD	18	20	6	27	18	20	22	48	15	27	45	34	12	19	36	36
CoV	0.12	0.09	0.03	0.08	0.10	0.07	0.07	0.13	0.08	0.10	0.10	0.09	0.06	0.08	0.08	0.08
Beech	132	235	315	291	144	268	304	324	174	235	347	391	132	253	359	353
PU	174	209	256	300	135	247	265	264	127	194	356	324	177	180	332	450
	165	229	281	277	147	215	271	321	127	188	386	383	138	182	350	450
	124	209	306	280	132	201	277	262	168	241	353	324	171	185	306	365
	168	227	262	238	147	262	318	323	122	209	309	377	153	247	347	362
Average	152	222	284	277	141	239	287	299	143	214	350	360	154	209	339	396
STD	23	12	26	24	7	29	23	33	25	24	27	33	19	37	21	50
CoV	0.15	0.06	0.09	0.09	0.05	0.12	0.08	0.11	0.18	0.11	0.08	0.09	0.13	0.18	0.06	0.13
Pine	79	118	155	171	88	147	162	230	153	215	191	306	168	227	330	368
PVA	109	115	191	162	109	141	227	206	144	180	162	265	156	288	409	309
	79	132	156	171	129	153	132	238	118	177	206	265	221	268	283	397
	82	103	168	132	141	159	197	206	144	171	221	221	185	206	280	368
	88	144	185	172	112	197	162	202	171	171	188	265	227	227	341	307
Average	88	122	171	162	116	160	176	216	146	183	194	264	191	243	328	350
STD	12	16	17	17	20	22	36	16	19	18	22	30	31	34	53	40
CoV	0.14	0.13	0.10	0.10	0.18	0.14	0.21	0.08	0.13	0.10	0.11	0.11	0.16	0.14	0.16	0.11
Pine	112	136	174	188	127	180	203	215	177	200	215	309	221	259	353	418
PU	132	124	168	171	121	200	197	285	178	227	274	250	206	268	341	456
	103	139	165	177	144	182	182	264	177	221	206	262	185	191	333	441
	109	93	177	168	162	150	197	271	187	192	244	221	177	230	341	380
	97	164	156	165	118	150	238	244	188	259	247	265	185	212	359	409
Average	111	131	168	174	134	172	204	256	181	220	237	261	195	232	346	421
STD	13	26	8	9	19	22	21	27	6	26	27	32	18	32	11	30
CoV	0.12	0.20	0.05	0.05	0.14	0.13	0.10	0.11	0.03	0.12	0.11	0.12	0.09	0.14	0.03	0.07

PVA, polyvinyl acetate; PU, polyurethane.

Table 3. Individual test/predicted ratios for T-shaped mortise and tenon joints.

	Tenon width (mm)															
	30				40				50				60			
	Tenon length (mm)															
	20	30	40	45	20	30	40	45	20	30	40	45	20	30	40	45
	Ratio—test value/predicted value															
Beech	1.42	1.18	0.89	1.12	1.03	1.45	1.15	1.10	1.04	0.95	1.43	0.85	1.09	0.80	1.03	0.87
PVA	1.30	1.05	0.91	1.03	1.34	1.23	1.00	1.10	1.20	1.08	1.20	0.78	1.15	0.80	1.19	0.97
	1.07	1.34	0.87	1.12	1.18	1.25	0.98	1.28	1.23	0.95	1.35	0.93	0.98	0.83	1.27	0.87
	1.09	1.27	0.88	1.26	1.23	1.33	1.05	0.92	1.07	1.17	1.21	0.97	1.09	0.92	1.17	0.87
	1.21	1.17	0.85	1.24	1.09	1.27	1.10	0.97	1.19	1.13	1.49	0.97	1.06	0.94	1.25	1.05
Beech																
PU	1.14	1.36	1.36	1.12	1.00	1.24	1.24	1.06	1.00	1.04	0.94	1.04	1.04	0.71	0.91	0.85
	1.50	1.20	1.11	1.15	0.94	1.14	1.14	0.92	0.82	0.76	0.77	1.06	0.86	0.95	0.65	1.08
	1.42	1.32	1.21	1.06	1.02	0.99	0.99	0.94	0.99	0.76	0.75	1.15	1.02	0.74	0.65	1.08
	1.07	1.20	1.32	1.07	0.92	0.93	0.93	0.96	0.81	1.00	0.96	1.05	0.86	0.92	0.67	0.88
	1.45	1.30	1.13	0.92	1.02	1.21	1.21	1.10	1.00	0.73	0.83	0.92	1.00	0.83	0.89	0.87
Pine	0.87	0.86	0.85	0.83	0.77	0.86	0.71	0.89	1.15	1.08	0.72	1.03	1.14	1.03	1.12	1.11
PVA	1.19	0.83	1.04	0.78	0.95	0.82	0.99	0.80	1.09	0.90	0.61	0.89	1.06	1.31	1.39	0.94
	0.87	0.96	0.85	0.83	1.13	0.89	0.58	0.93	0.89	0.89	0.78	0.89	1.50	1.22	0.96	1.20
	0.90	0.75	0.91	0.64	1.24	0.93	0.86	0.80	1.09	0.86	0.83	0.74	1.26	0.94	0.95	1.11
	0.96	1.05	1.01	0.83	0.98	1.15	0.71	0.79	1.29	0.86	0.71	0.89	1.54	1.03	1.16	0.93
Pine	1.28	1.04	1.00	0.96	1.17	1.10	0.94	0.88	1.40	1.06	0.85	1.09	1.58	1.24	1.27	1.33
PU	1.52	0.95	0.96	0.87	1.11	1.23	0.91	1.17	1.41	1.20	1.09	0.88	1.48	1.28	1.22	1.45
	1.18	1.06	0.95	0.90	1.33	1.12	0.84	1.08	1.40	1.17	0.82	0.92	1.33	0.91	1.19	1.41
	1.25	0.71	1.01	0.85	1.49	0.92	0.91	1.11	1.48	1.02	0.97	0.78	1.27	1.10	1.22	1.21
	1.11	1.25	0.89	0.84	1.09	0.92	1.10	1.00	1.50	1.37	0.98	0.93	1.33	1.01	1.29	1.30

PVA, polyvinyl acetate; PU, polyurethane.

number of joints might be expected to have capacities less than the chosen level, whereas few joints would be expected to have less capacity than the LTL at the 90%|90% confidence|proportion level; however, LTLs at the 75|90 and 90|75 confidence|proportion levels also were included in the study to provide added information.

RESULTS

Test/Estimated Ratios

The test-capacity/estimated-capacity ratios for the four broad specimen groups (pine/polyvinyl acetate [PVA], pine/polyurethane [PU], beech/PVA, and beech/PU) are given in Table 3.

The average value of the 320 ratios was 1.045 with a standard deviation of 0.199. Of these ratios, 173 were greater than 1.0, whereas 147 were less than 1.0. The minimum ratio was 0.580 and the highest ratio was 1.583.

Distribution of ratios in terms of the above four categories was as follows: In the case of the beech|PVA configuration, 26 ratios were less than and 54 were greater than 1.0. Corresponding statistics for the beech|PU configuration, the pine|PVA configuration, and the pine|PU configuration were 42 less vs 38 greater than 1.0, 52 less vs 28 greater than 1.0, and 27 less vs 53 greater than 1.0, respectively.

Equation (1), therefore, overestimates the capacity of 26 (32.5%) and underestimates the capacity of 54 (67.5%) of the 80 beech|PVA joints. Likewise, equation (1) overestimates the capacity of 42 (52.5%) and underestimates the capacity of 38 (47.5%) of the 80 beech|PU joints, overestimates the capacity of 52 (65%) and underestimates the capacity of

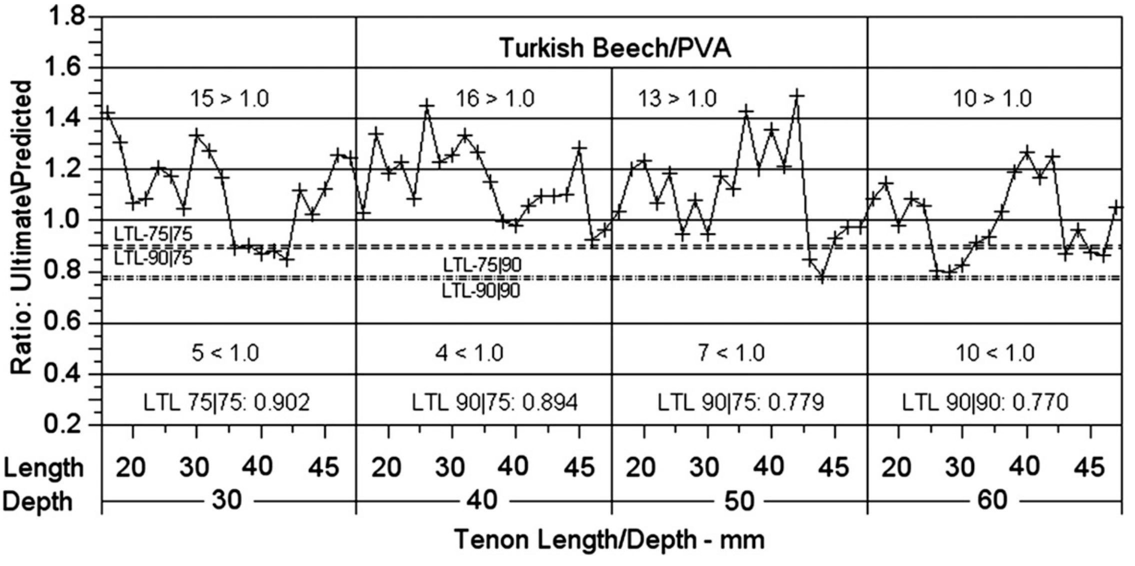

Figure 3. Graph showing individual ratios of test/estimated values for Turkish beech specimens with polyvinyl acetate (PVA) adhesive.

28 (35%) of the 80 pine|PVA joints, and overestimates the capacity of 27 (33.8%) and underestimates the capacity of 53 (66.3%) of the 80 pine|PU joints.

To better visualize the distribution of the individual ratios above and below the average, values of the ratios along with the average are illustrated in Figs 3-6.

LTLS FOR RATIOS

The tolerance factors, k, for 320 specimens at the 75|75, 90|75, 75|90, and 90|90 confidence|

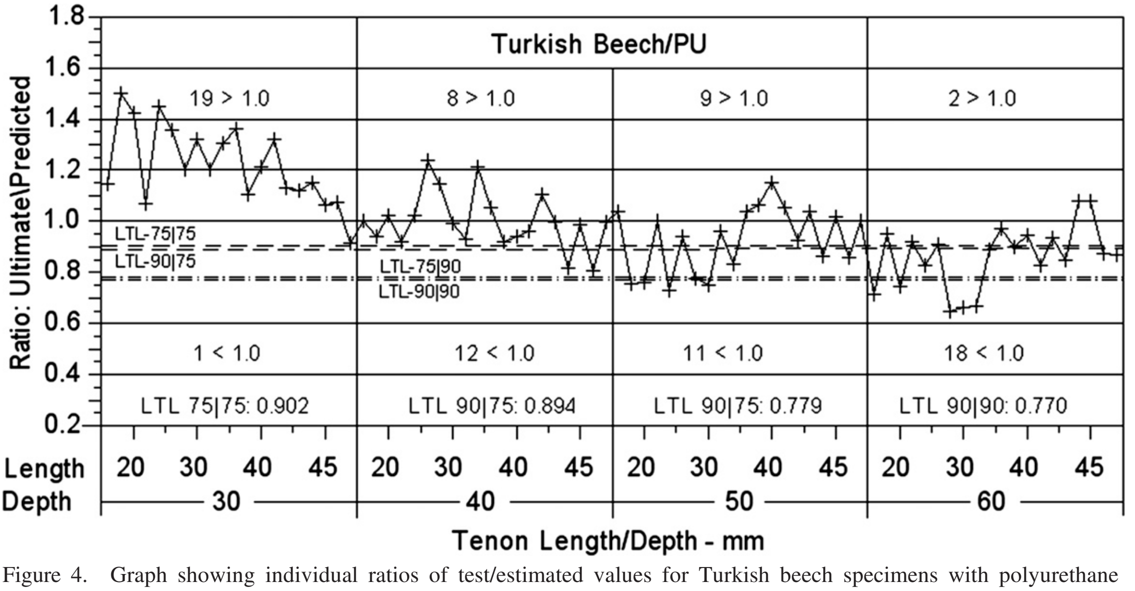

Figure 4. Graph showing individual ratios of test/estimated values for Turkish beech specimens with polyurethane (PU) adhesive.

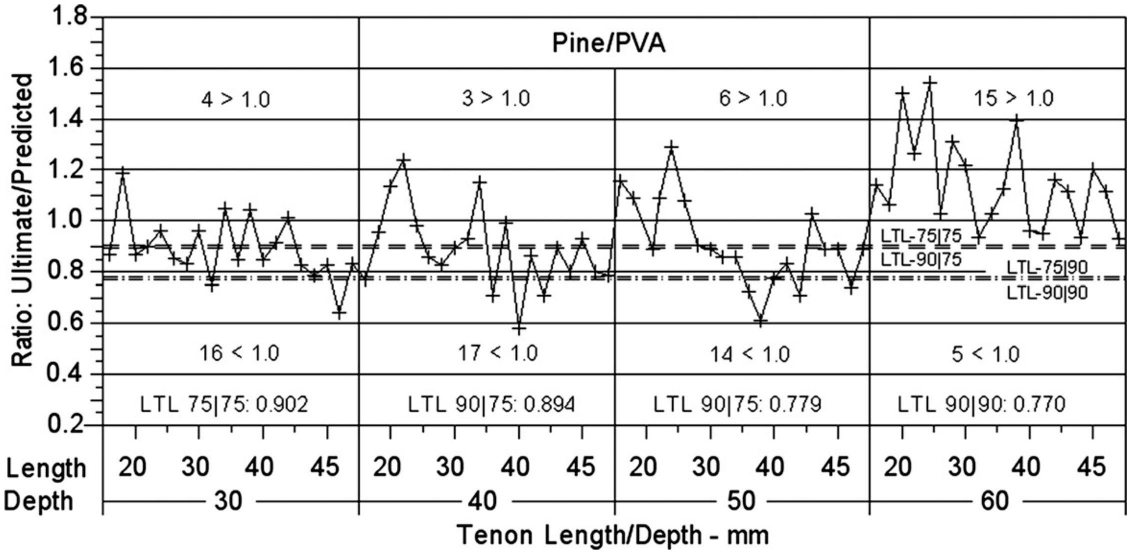

Figure 5. Graph showing individual ratios of test/estimated values for pine specimens with a polyvinyl acetate (PVA) adhesive.

proportion levels were 0.717, 0.756, 1.335, and 1.383, respectively.

At the 75 × 75 confidence–proportion level, the LTL for the entire collection of transformed data (ie specimen capacity/estimated capacity) using the *k* factor for 320 specimens of 0.717 was

$$\begin{aligned} \text{LTL}\,(75|75) &= 1.045 - 0.717 \times 0.199 \\ &= 0.902. \end{aligned} \tag{3}$$

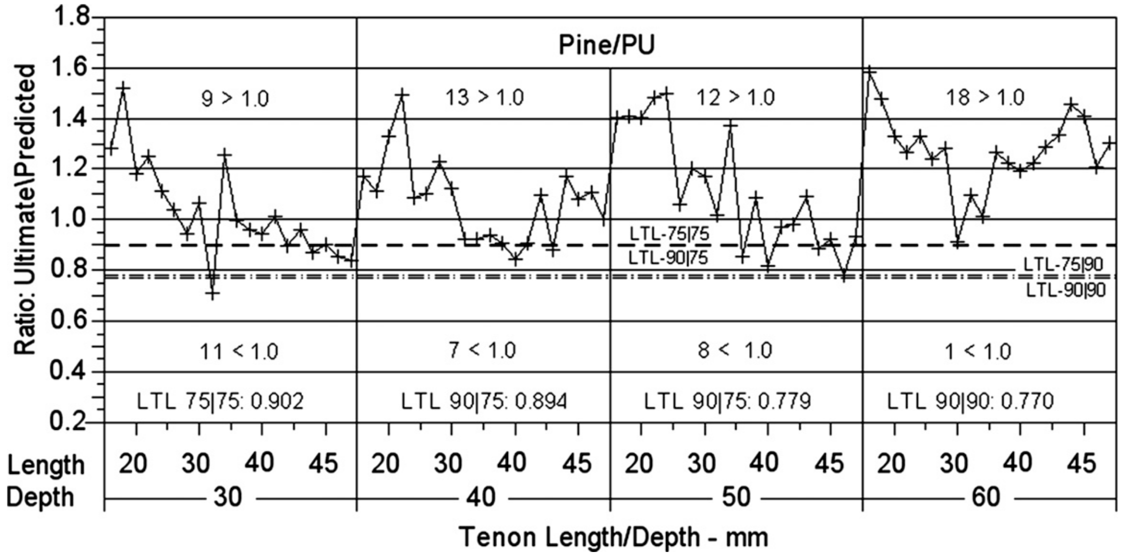

Figure 6. Graph showing individual ratios of test/estimated values for Pine specimens with a polyurethane (PU) adhesive.

Thus, the LTL for the transformed data at this confidence–proportion level amounted to 0.902 of the average of the ratios (ie 1.0)—thus, the corresponding LTL for equation (1) would amount to 90.2% of the estimated values.

Referring to figs 3-6, it can be seen that at the 75|75 confidence|proportion level, 84 ratios (26.3%) were less than the LTL of 0.902. The distribution of these ratios below the LTL was as follows: 53 (16.6%) had values that were within the range of 90-100% of the LTL, 20 (6.3%) within the range of 80-90%, 9 (2.8%) within the range of 70-80%, and 2 (0.56 %) within the range of 60-70% of LTL.

At the highest confidence|proportion level examined, ie the 90|90 level, the tolerance factor, *k*, is 1.383 so that the corresponding LTL is

$$\begin{aligned} \text{LTL}\ (90|90) &= 1.045 - 1.383 \times 0.199 \\ &= 0.770. \end{aligned} \tag{4}$$

Thus, the LTL for the ratios at this confidence|proportion level amounted to 0.770 of the average value, so that the corresponding LTL for equation (1) would amount to 77.0% of the estimated values.

At this (90|90) confidence|proportion level (figs 3-6), only 19 (5.9%) specimens had ratios less than the LTL of 0.770. The distribution of ratios below the 90|90 LTL was as follows: 13 (4.1%) had values that were within the range of 90-100% of the LTL, 4 (1.3%) within the range of 80-90%, and 2 (0.6%) within the range of 70-80%. Thus, there is a substantial reduction in the number of ratios below the LTL but at a substantial reduction of 17.2% in the value of the LTL (0.770 vs 0.902). This result emphasizes the importance of determining what percentage of failure—if any—is acceptable along with what level of confidence is appropriate.

As can be seen in figs 3-6, at the intermediate 90|75 and 75|90 LTL levels that were examined, the 90|75 LTL amounts to 0.894, which is only slightly less (0.9%) than the 75|75 LTL of 0.902. The distribution below the 90|75 level was as follows: 51 (15.9%) were within the range of 90-100, 18 (5.6%) were within the range 80-90, 9 (2.8%) were within the range of 70-80, and 2 (.63%) were within the range of 60-70%

Likewise, the 75|90 LTL amounts to 0.779, which is only slightly greater (1.2%) than the 90|90 LTL of 0.770. The distribution of values below the 75|90 LTL level was as follows: 16 (5.0%) were within the range of 90-100, 4 (1.3%) were within the range of 80-90, and 2 (6.3%) were within the range of 70-80% of the LTL.

Thus, the distribution of ratios below the LTLs for these confidence|proportion levels is essentially the same as for the 75|75 and 90|90 ratios.

Finally, it should be noted that in considering the broader application of these techniques, it should be noted that the material from which these joints were constructed was presumably largely defect free, and the specimens were constructed under closely controlled conditions. Had the specimens been constructed under less closely controlled conditions, presumably with accompanying increases in standard deviation, the LTLs would have been appropriately lower.

CONCLUSIONS

Use of statistical LTL techniques provides a means of rationally determining joint capacity design values as a percentage of the values estimated by nonlinear regression expressions fitted to the test data. Specifically, LTL values based on the mean and standard deviation of the ratios formed by dividing individual test values by their corresponding estimated values provides the information needed to calculate the percentage of specimens that might be expected to have less capacity than specified percentages of the estimated values along with specified degrees of confidence in those calculations.

The results of this study alone do not provide definitive answers to the question of what are appropriate confidence–proportion levels to be used in deriving joint capacity design values from nonlinear regression expressions fitted to results of test data. Determination of widely applicable rational design values for mortise and tenon joints based on statistical LTLs will require extensive sampling of data related to the capacity of joints constructed under a variety of quality control scenarios, particularly under "normal" manufacturing conditions as well as laboratory conditions.

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