OPEN ACCESS
Fractals, Vol. 28, No. 8 (2020) 2040040 (16 pages)

DOI: 10.1142/S0218348X2040040X

ITERATIVE METHOD APPLIED TO THE FRACTIONAL NONLINEAR SYSTEMS ARISING IN THERMOELASTICITY WITH MITTAG-LEFFLER KERNEL

WEI GAO[*,‖], P. VEERESHA[†,**], D. G. PRAKASHA[‡,††], BILGIN SENEL[§,‡‡]
and HACI MEHMET BASKONUS[¶,§§]

[*]*School of Information Science and Technology, Yunnan Normal University*
Yunnan, P. R. China

[†]*Department of Mathematics, Karnatak University, Dharwad 580003, India*

[‡]*Department of Mathematics, Davangere University, Shivagangothri*
Davangere 577007, India

[§]*Fethiye Faculty of Business Administration, Mugla Sitki Kocman University*
Mugla, Turkey

[¶]*Department of Mathematics and Science Education, Harran University*
Sanliurfa, Turkey

[‖]*gaowei@ynnu.edu.cn*
[**]*viru0913@gmail.com*
[††]*prakashadg@gmail.com*
[‡‡]*senelbilgin@gmail.com*
[§§]*hmbaskonus@gmail.com*

Received December 19, 2019
Accepted February 23, 2020
Published September 18, 2020

[‖]Corresponding author.

Abstract

In this paper, we study on the numerical solution of fractional nonlinear system of equations representing the one-dimensional Cauchy problem arising in thermoelasticity. The proposed technique is graceful amalgamations of Laplace transform technique with q-homotopy analysis scheme and fractional derivative defined with Atangana–Baleanu (AB) operator. The fixed-point hypothesis is considered in order to demonstrate the existence and uniqueness of the obtained solution for the proposed fractional order model. In order to illustrate and validate the efficiency of the future technique, we consider three different cases and analyzed the projected model in terms of fractional order. Moreover, the physical behavior of the obtained solution has been captured in terms of plots for diverse fractional order, and the numerical simulation is demonstrated to ensure the exactness. The obtained results elucidate that the proposed scheme is easy to implement, highly methodical as well as accurate to analyze the behavior of coupled nonlinear differential equations of arbitrary order arisen in the connected areas of science and engineering.

Keywords: Laplace Transform; Atangana–Baleanu Derivative; q-Homotopy Analysis Method; Thermoelasticity; Fixed Point Theorem.

1. INTRODUCTION

Fractional calculus (FC) was originated in Newton's time, but lately it fascinated the attention of many scholars. From the last 30 years, the most intriguing leaps in scientific and engineering applications have been found within the framework of FC. The concept of fractional derivative has been industrialized due to the complexities associated with heterogeneities phenomenon. The fractional differential operators are capable to capture the behavior of multifaceted media having diffusion process. It has been a very essential tool, and many problems can be illustrated more conveniently and more accurately with differential equations having arbitrary order. Due to the swift development of mathematical techniques with computer software, many researchers started to work on generalized calculus to present their viewpoints while analyzing many complex phenomena.

Numerous pioneering directions are prescribed for the diverse definitions of FC by many senior researchers and which prearranged the foundation.[1–6] Calculus with fractional order is associated to practical ventures and it extensively employed to nanotechnology,[7] optics,[8] human diseases,[9] chaos theory,[10] and other areas.[11–14] The numerical and analytical solutions for these equations illustrating these models have an important role in portraying nature of nonlinear problems ascends in connected areas of science. Many physicists and mathematicians are magnetized by the study of interesting properties of materials like elasticity, thermal conductivity, malleability and hardenability, and many others. The study of properties of materials, such as thermal conductivity and its stresses or elasticity and temperature, is known as thermoelasticity. Recently, the study and analysis of these concepts are fascinating many researchers associated with diverse areas connected to mathematics. The inevitability of irrational physical behavior depiction of solid bodies by elastic deformations obtained with thermal stresses inspired the more prominent physicists and mathematicians as well as engineers.[15,16]

In the present investigation, the nonlinear system of equations representing the one-dimensional Cauchy problem arising in thermoelasticity of the form is[17,18]

$$\begin{aligned} &u_{tt} - a\left(u_x,\, v\right) u_{xx} + b\left(u_x,\, v\right) v_x \\ &\quad = f_1\left(x,t\right), \\ &c\left(u_x,\, v\right) v_t + b\left(u_x,\, v\right) u_{xt} - d\left(v\right) v_{xx} \\ &\quad = f_2\left(x,t\right), \end{aligned} \tag{1}$$

where u and v are, respectively, displacement and temperature difference, $a(u_x, v)$, $b(u_x,\, v)$, $c(u_x,\, v)$, $d(v)$, $f_1(x,t)$, and $f_2\,(x,t)$ are specified smooth functions. The considered nonlinear coupled problem recently fascinated the attention of researchers from different areas of science. Since system (1) plays a significant role in portraying several nonlinear phenomena and also which are the overviews of diverse complex problems. Many authors find and analyzed the solution using analytical as well as numerical schemes; for instance, authors in

Ref. 18 illustrated the numerical solution for considered coupled system with the aid of variational iteration algorithm. The asymptotic stability, global existence, and uniqueness have been illustrated in Ref. 19. The author in Ref. 20 hired Laplace decomposition technique in order to find the singular and nonsingular solutions for coupled system describing the physical behavior of thermoelasticity of the materials. The authors in Ref. 21 find the numerical solution for system (1) and presented some interesting results. The Adomian decomposition scheme is applied by the authors in Ref. 22 to find the numerical solution for the cited model.

In the present scenario, many important and nonlinear models are methodically and effectively analyzed with the help of FC. There have been diverse definitions suggested by many senior research scholars, for instance, Riemann, Liouville, Caputo, and Fabrizio. However, these definitions have their own limitations. The Riemann–Liouville derivative is unable to explain the importance of the initial conditions; the Caputo derivative has overcome this shortcoming but is impotent to explain the singular kernel of the phenomena. Later, in 2015 Caputo and Fabrizio defeated the above obliges,[23] and many researchers considered this derivative in order to analyze and find the solution for diverse classes of nonlinear complex problems. But some issues were pointed out in CF derivative, like nonsingular kernel and nonlocal, these properties are very essential in describing the physical behavior and nature of the nonlinear problems. In 2016, Atangana and Baleanu introduced and natured the novel fractional derivative, namely AB derivative. AB derivative defined with the aid of Mittag-Leffler functions.[24] This fractional derivative buried all the above-cited issues and help us to understand the natural phenomena in the systematic and effective way.

Recently, many mathematicians and physicists developed very effective and more accurate methods in order to find and analyze the solution for complex and nonlinear problems arising in science and engineering. In connection with this, the homotopy analysis method (HAM) was proposed by Chinese mathematician *Liao Shijun*.[25,26] HAM has been profitably and effectively applied to study the behavior of nonlinear problems without perturbation or linearization. But, for computational work, HAM requires huge memory of computers and also time. Hence, there is an essence of the amalgamation of this method with well-known transform techniques.

In the present investigation, we put an effort to find and analyze behavior of solution obtained for the system of equations presented in Eq. (1) with fractional order of the form

$$\begin{aligned} &{}_{a}^{ABC}D_t^{\alpha+1}u\,(x,t) - a\,(u_x,\, v)\,u_{xx} \\ &\quad + b\,(u_x,\, v)\,v_x = f_1\,(x,t)\,, \quad 0<\alpha\le 1, \\ &c\,(u_x,\, v)\,{}_{a}^{ABC}D_t^{\beta}v\,(x,t) + b\,(u_x,\, v)\,u_{xt} \\ &\quad - d\,(v)\,v_{xx} = f_2\,(x,t)\,,\ 0<\beta\le 1, \end{aligned} \tag{2}$$

where α and β are fractional orders of the system, defined with AB fractional operator.

The fractional order is introduced in order to incorporate the memory effects and hereditary consequence in the system and these properties aid us to capture essential physical properties of the complex problems. The future algorithm is the combination of q-HAM with LT.[27] Since q-HATM is an improved scheme of HAM, it does not require discretization, perturbation, or linearization. Recently, due to its reliability and efficacy, the considered method is exceptionally applied by many researchers to understand physical behavior diverse classes of complex problems.[28–34] The proposed method offers us with more freedom to consider diverse class of initial guesses and the equation type complex as well as nonlinear problems; because of this, the complex NDEs can be directly solved. The future method offers simple algorithm to evaluate the solution and it is natured by the homotopy and axillary parameters, which provide the rapid convergence in the obtained solution for nonlinear portion of the given problem. Meanwhile, it has prodigious generality because it plausibly contains the results obtained by many algorithms like q-HAM, HPM, ADM, and some other traditional techniques. The considered method can preserve great accuracy while decreasing the computational time and work in comparison with other methods.

2. PRELIMINARIES

Recently, many authors have considered various derivatives to analyze a diverse class of models in comparison with classical order.[35–47] In this section, we define basic notion of AB derivatives and integrals.[24]

Definition 1. The fractional Atangana–Baleanu–Caputo derivative for a function $f \in H^1(a,b)(b > a, \alpha \in [0,1])$ is presented as follows:

$$ {}_{a}^{ABC}D_t^{\alpha}\left(f\left(t\right)\right) = \frac{B\left[\alpha\right]}{1-\alpha}\int_a^t f'\left(\vartheta\right) E_{\alpha}\left[\alpha\frac{\left(t-\vartheta\right)^{\alpha}}{\alpha-1}\right] d\vartheta, \tag{3}$$

where $B\left[\alpha\right]$ is a normalization function such that $B(0) = B(1) = 1$.

Definition 2. The AB derivative of fractional order for a function $f \in H^1\left(a,\, b\right),\, b > a,\ \alpha \in [0,\, 1]$ in Riemann–Liouville sense is presented as follows:

$$ {}_{a}^{ABR}D_t^{\alpha}\left(f\left(t\right)\right) = \frac{B\left[\alpha\right]}{1-\alpha}\frac{d}{dt}\int_a^t f\left(\vartheta\right) E_{\alpha}\left[\alpha\frac{\left(t-\vartheta\right)^{\alpha}}{\alpha-1}\right] d\vartheta. \tag{4}$$

Definition 3. The fractional AB integral related to the nonlocal kernel is defined by

$$ {}_{a}^{AB}I_t^{\alpha}\left(f\left(t\right)\right) = \frac{1-\alpha}{B\left[\alpha\right]}f\left(t\right) + \frac{\alpha}{B\left[\alpha\right]\Gamma\left(\alpha\right)} \times \int_a^t f\left(\vartheta\right)\left(t-\vartheta\right)^{\alpha-1} d\vartheta. \tag{5}$$

Definition 4. The Laplace transform (LT) of AB derivative is defined by

$$ L\left[{}_{0}^{ABR}D_t^{\alpha}\left(f\left(t\right)\right)\right] = \frac{B\left[\alpha\right]}{1-\alpha}\frac{s^{\alpha}L\left[f\left(t\right)\right] - s^{\alpha-1}f\left(0\right)}{s^{\alpha} + \left(\alpha/(1-\alpha)\right)}. \tag{6}$$

Theorem 1. *The following Lipschitz conditions, respectively, hold true for both Riemann–Liouville and AB derivatives defined in Eqs.* (3) *and* (4)[24]:

$$ \left\|{}_{a}^{ABC}D_t^{\alpha}f_1\left(t\right) - {}_{a}^{ABC}D_t^{\alpha}f_2\left(t\right)\right\| < K_1\left\|f_1\left(x\right) - f_2\left(x\right)\right\| \tag{7}$$

and

$$ \left\|{}_{a}^{ABC}D_t^{\alpha}f_1\left(t\right) - {}_{a}^{ABC}D_t^{\alpha}f_2\left(t\right)\right\| < K_2\|f_1(x). - f_2(x)\|. \tag{8}$$

Theorem 2. *The time-fractional differential equation* ${}_{a}^{ABC}D_t^{\alpha}f_1\left(t\right) = s\left(t\right)$ *has a unique solution and which is defined as*[24]

$$ f\left(t\right) = \frac{\left(1-\alpha\right)}{B\left[\alpha\right]}s\left(t\right) + \frac{\alpha}{B\left[\alpha\right]\Gamma\left(\alpha\right)} \times \int_0^t s\left(\varsigma\right)\left(t-\varsigma\right)^{\alpha-1} d\varsigma. \tag{9}$$

3. FUNDAMENTAL IDEA OF THE PROPOSED SCHEME

In this segment, we consider the arbitrary order differential equation in order to demonstrate the fundamental solution procedure of the proposed scheme:

$$ {}_{a}^{ABC}D_t^{\alpha}v\left(x,t\right) + R\,v\left(x,t\right) + N\,v\left(x,t\right) = f\left(x,t\right), \quad n-1 < \alpha \le n, \tag{10}$$

with the initial condition

$$ v\left(x,0\right) = g\left(x\right), \tag{11}$$

where ${}_{a}^{ABC}D_t^{\alpha}v\left(x,t\right)$ symbolize the AB derivative of $v\left(x,t\right) f, \left(x,t\right)$ signifies the source term, R and N, respectively, denote the linear and nonlinear differential operator. On using the LT on Eq. (10), we have after simplification

$$ L\left[v\left(x,t\right)\right] - \frac{g\left(x\right)}{s} + \frac{1}{B\left[\alpha\right]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right) \times \left\{L\left[Rv\left(x,t\right)\right] + L\left[N\,v\left(x,t\right)\right] - L\left[f\left(x,t\right)\right]\right\} = 0. \tag{12}$$

The nonlinear operator is defined as follows:

$$ N\left[\varphi\left(x,t;q\right)\right] = L\left[\varphi\left(x,t;q\right)\right] - \frac{g\left(x\right)}{s} + \frac{1}{B\left[\alpha\right]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right) \times \left\{L\left[R\,\varphi\left(x,t;q\right)\right] + L\left[N\varphi\left(x,t;q\right)\right] - L\left[f\left(x,t\right)\right]\right\}. \tag{13}$$

Here, $\varphi(x,\, t;q)$ is the real-valued function with respect to xt and $\left(q \in \left[0, \frac{1}{n}\right]\right)$. Now, we define a homotopy as follows:

$$ \left(1-nq\right) L\left[\varphi\left(x,\ t;q\right) - v_0\left(x,t\right)\right] = \hbar q N\left[\varphi\left(x,\ t;q\right)\right], \tag{14}$$

where L is signifying LT, $q \in \left[0, \frac{1}{n}\right]$ $(n \ge 1)$ is the embedding parameter, and $\hbar \neq$ is an auxiliary parameter. For $q = 0$ and $q = \frac{1}{n}$, the results given below hold true

$$ \varphi\left(x,t;0\right) = v_0\left(x,t\right),\ \varphi\left(x,t;\frac{1}{n}\right) = v\left(x,t\right). \tag{15}$$

Thus, by intensifying q from to $\frac{1}{n}$, the solution $\varphi(x,\, t;q)$ varies from $v_0(x,t)$ to $v(x,t)$. By using the Taylor theorem near to q, we define $\varphi\left(x,t;q\right)$ in series form and then we get

$$ \varphi\left(x,t;q\right) = v_0\left(x,t\right) + \sum_{m=1}^{\infty} v_m\left(x,t\right) q^m, \tag{16}$$

where

$$v_m(x,t) = \frac{1}{m!}\left.\frac{\partial^m \varphi(x,\ t;q)}{\partial q^m}\right|_{q=0}. \quad (17)$$

The series (14) converges at $q = \frac{1}{n}$ for the proper choice of $v_0(x,t)$, n and $\hbar$. Then

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t)\left(\frac{1}{n}\right)^m. \quad (18)$$

Now, m-times differentiating Eq. (15) with q and later dividing by $m!$ and then putting $q = 0$, we obtain

$$L[v_m(x,t) - k_m v_{m-1}(x,t)] = \hbar R_m(\vec{v}_{m-1}), \quad (19)$$

where the vectors are defined as

$$\vec{v}_m = \{v_0(x,t), v_1(x,t), \ldots, v_m(x,t)\}. \quad (20)$$

On applying inverse LT on Eq. (19), one can get

$$v_m(x,t) = k_m v_{m-1}(x,t) + \hbar L^{-1}[R_m(\vec{v}_{m-1})], \quad (21)$$

where

$$\begin{aligned} R_m(\vec{v}_{m-1}) = {} & L[v_{m-1}(x,t)] \\ & - \left(1 - \frac{k_m}{n}\right)\left(\frac{g(x)}{s} + \frac{1}{B[\alpha]}\right. \\ & \left.\times\left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[f(x,t)]\right) \\ & + \frac{1}{B[\alpha]}\left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) \\ & \times L[Rv_{m-1} + H_{m-1}], \end{aligned} \quad (22)$$

and

$$k_m = \begin{cases} 0, & m \leq 1, \\ n, & m > 1. \end{cases} \quad (23)$$

In Eq. (22), H_m signifies homotopy polynomial and presented as follows:

$$H_m = \frac{1}{m!}\left[\frac{\partial^m \varphi(x,\ t;q)}{\partial q^m}\right]_{q=0} \text{ and } \quad (24)$$
$$\varphi(x,\ t;q) = \varphi_0 + q\varphi_1 + q^2\varphi_2 + \cdots.$$

By the aid of Eqs. (21) and (22), one can get

$$\begin{aligned} v_m(x,t) = {} & (k_m + \hbar)\, v_{m-1}(x,t) \\ & - \left(1 - \frac{k_m}{n}\right) L^{-1}\left(\frac{g(x)}{s}\right. \\ & \left. + \frac{1}{B[\alpha]}\left(1 - \alpha + \frac{\alpha}{s^\alpha}\right) L[f(x,t)]\right) \\ & + \hbar L^{-1}\left\{\frac{1}{B[\alpha]}\left(1 - \alpha + \frac{\alpha}{s^\alpha}\right)\right. \\ & \left.\times L[Rv_{m-1} + H_{m-1}]\right\}. \end{aligned} \quad (25)$$

Using Eq. (25), one can get the series of $v_m(x,t)$. Lastly, the series q-HATM solution is defined as

$$v(x,t) = v_0(x,t) + \sum_{m=1}^{\infty} v_m(x,t)\left(\frac{1}{n}\right)^m. \quad (26)$$

4. SOLUTION FOR FKGZ EQUATIONS

In order to present the solution procedure and efficiency of the future scheme, in this segment, we consider KGZ equations of fractional order with two distinct cases. Further by the help of obtained results, we made an attempt to capture the behavior of q-HATM solution for different fractional orders. By substituting $a(u_x,\ v) = 2 - u_x v$, $b(u_x,\ v) = 2 + u_x v$, $c(u_x,\ v) = 1$, $d(v) = v$ in Eq. (2), we have

$$\begin{aligned} & {}_{a}^{ABC}D_t^{\alpha+1} u(x,t) - (2 - vu_x)\, u_{xx} \\ & \quad + (2 + vu_x)\, v_x = f_1(x,t), \\ & {}_{a}^{ABC}D_t^{\beta} v(x,t) \\ & \quad + (2 + vu_x)\, u_{xt} - vv_{xx} = f_2(x,t), \end{aligned} \quad (27)$$

with initial conditions

$$\begin{aligned} & u(x,0) = v(x,0) = g(x), \\ & u_t(x,0) = 0. \end{aligned} \quad (28)$$

Taking LT on Eq. (27) and then using Eq. (28) and by the help of results derived in Ref. 48, we get

$$\begin{aligned} L[u(x,t)] = {} & \frac{1}{s}(g(x)) - \frac{1}{B[\alpha]}\left(\frac{s^\alpha + \alpha(1 - s^\alpha)}{s^{\alpha+1}}\right) \\ & \times L\left\{\left(2 - v\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2}\right. \\ & \left. - \left(2 + v\frac{\partial u}{\partial x}\right)\frac{\partial v}{\partial x} + f_1\right\}, \end{aligned} \quad (29)$$

$$\begin{aligned} L[v(x,t)] = {} & \frac{1}{s}(g(x)) + \frac{1}{B[\beta]}\left(\frac{s^\beta - \beta(1 - s^\beta)}{s^\beta}\right) \\ & \times L\left\{\left(2 + v\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x \partial t}\right. \\ & \left. - v\frac{\partial^2 v}{\partial x^2} - f_2\right\}. \end{aligned}$$

The nonlinear operator N is presented with the help of future algorithm[48] as below:

$$\begin{aligned}
&N^1\left[\varphi_1(x,t;q),\,\varphi_2(x,t;q)\right]=L\left[\varphi_1(x,t;q)\right]\\
&\quad-\frac{1}{s}\left(g(x)\right)+\frac{1}{B[\alpha]}\left(\frac{s^\alpha+\alpha\left(1-s^\alpha\right)}{s^{\alpha+1}}\right)\\
&\quad\times L\left(2-\varphi_2\frac{\partial\varphi_1}{\partial x}\right)\frac{\partial^2\varphi_1}{\partial x^2}-\left(2+\varphi_2\frac{\partial\varphi_1}{\partial x}\right)\\
&\quad\times\frac{\partial\varphi_2}{\partial x}+f_1,\\
&N^2\left[\varphi_1(x,t;q),\,\varphi_2(x,t;q)\right]=L\left[\varphi_2(x,t;q)\right]\\
&\quad-\frac{1}{s}\left(g(x)\right)-\frac{1}{B[\beta]}\left(\frac{s^\beta-\beta\left(1-s^\beta\right)}{s^\beta}\right)\\
&\quad\times L\left(2+\varphi_2\frac{\partial\varphi_1}{\partial x}\right)\frac{\partial^2\varphi_1}{\partial x\partial t}-\varphi_2\frac{\partial^2\varphi_2}{\partial x^2}-f_2.
\end{aligned}\tag{30}$$

The deformation equation of mth order by the help of q-HATM at $H(x,t)=1$ is given as follows:

$$\begin{aligned}
L\left[u_m(x,t)-k_m u_{m-1}(x,t)\right]&=\hbar R_{1,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right],\\
L\left[v_m(x,t)-k_m v_{m-1}(x,t)\right]&=\hbar R_{2,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right],
\end{aligned}\tag{31}$$

where

$$\begin{aligned}
&R_{1,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right]=L\left[u_{m-1}(x,t)\right]\\
&\quad-\left(1-\frac{k_m}{n}\right)\left\{\frac{1}{s}\left(g(x)\right)\right\}\\
&\quad-\frac{1}{B[\alpha]}\left(\frac{s^\alpha+\alpha\left(1-s^\alpha\right)}{s^{\alpha+1}}\right)\\
&\quad\times L2\frac{\partial^2u_{m-1}}{\partial x^2}-\sum_{i=0}^{m-1}\sum_{j=0}^{i}v_j\frac{\partial u_{i-j}}{\partial x}\frac{\partial^2u_{m-1-i}}{\partial x^2}\\
&\quad-2\frac{\partial v_{m-1}}{\partial x}-\sum_{i=0}^{m-1}\sum_{j=0}^{i}v_j\frac{\partial u_{i-j}}{\partial x}\frac{\partial v_{m-1-i}}{\partial x}+f_1,
\end{aligned}\tag{32}$$

$$\begin{aligned}
&R_{2,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right]=L\left[v_{m-1}(x,t)\right]\\
&\quad+\left(1-\frac{k_m}{n}\right)\left\{\frac{1}{s}\left(g(x)\right)\right\}\\
&\quad+\frac{1}{B[\beta]}\left(\frac{s^\beta-\beta\left(1-s^\beta\right)}{s^\beta}\right)L2\frac{\partial^2u_{m-1}}{\partial x\partial t}\\
&\quad+\sum_{i=0}^{m-1}\sum_{j=0}^{i}v_j\frac{\partial u_{i-j}}{\partial x}\frac{\partial^2u_{m-1-i}}{\partial x\partial t}\\
&\quad-\sum_{i=0}^{m-1}v_i\frac{\partial^2v_{m-1-i}}{\partial x^2}-f_2.
\end{aligned}$$

On applying inverse LT on Eq. (31), it reduces to

$$\begin{aligned}
u_m(x,t)&=k_m u_{m-1}(x,t)\\
&\quad+\hbar L^{-1}\left\{R_{1,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right]\right\},\\
v_m(x,t)&=k_m v_{m-1}(x,t)\\
&\quad+\hbar L^{-1}\left\{R_{2,m}\left[\vec{u}_{m-1},\,\vec{v}_{m-1}\right]\right\}.
\end{aligned}\tag{33}$$

Now, by simplifying the above equations systematically, we can evaluate the terms of the series solution

$$\begin{aligned}
u(x,t)&=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t)\left(\frac{1}{n}\right)^m,\\
v(x,\,t)&=v_0(x,t)+\sum_{m=1}^{\infty}v_m(x,t)\left(\frac{1}{n}\right)^m.
\end{aligned}\tag{34}$$

5. EXISTENCE OF SOLUTION

Here, we considered the fixed-point theorem in order to demonstrate the existence of the solution for the proposed model. Since the considered model cited in the system (27) is nonlocal as well as complex; there are no particular algorithms or methods to evaluate the exact solutions. However, under some particular conditions, the existence of the solution assurances. Now, system (27) is considered as follows:

$$\begin{cases}{}_0^{ABC}D_t^\alpha\left[u(x,t)\right]=G_1(x,t,\,u),\\ {}_0^{ABC}D_t^\beta\left[v(x,t)\right]=G_2(x,t,v).\end{cases}\tag{35}$$

The foregoing system is transformed to the Volterra integral equation using Theorem 2 and is as follows:

$$\begin{cases}u(x,t)-u(x,0)=\dfrac{(1-\alpha)}{B(\alpha)}G_1(x,t,u)\\ \qquad+\dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\displaystyle\int_0^t G_1(x,\zeta,u)(t-\zeta)^{\alpha-1}d\zeta,\\ v(x,t)-v(x,0)=\dfrac{(1-\beta)}{B(\beta)}G_2(x,t,v)\\ \qquad+\dfrac{\beta}{B(\beta)\Gamma(\beta)}\displaystyle\int_0^t G_2(x,\zeta,v)(t-\zeta)^{\beta-1}d\zeta.\end{cases}\tag{36}$$

Theorem 3. *The kernel G_1 satisfies the Lipschitz condition and contraction if the condition $0\le\left(2\delta^2+\frac{1}{2}\lambda_2\delta(a+b)+\tau_2\left(2+\lambda_2\delta\right)+\xi_1\right)<1$ holds.*

Proof. In order to prove the required result, we consider the two functions u and u_1, then

$$\begin{aligned}
&\left\|G_1(x,t,\,u)-G_1(x,t,\,u_1)\right\|\\
&\quad=\left\|-2\frac{\partial^2}{\partial x^2}\left[u(x,t)-u(x,t_1)\right]+v\left(\frac{\partial}{\partial x}\left[u(x,t)\right.\right.\right.
\end{aligned}$$

$$-u\left(x,t_1\right)]\frac{\partial^2}{\partial x^2}\left[u(x,t)-u\left(x,t_1\right)\right]\Big)$$
$$+2\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial x}\left(\frac{\partial}{\partial x}\left[u\left(x,t\right)-u\left(x,t_1\right)\right]\right)-f_1\Big\|$$
$$=\Big\|-2\frac{\partial^2}{\partial x^2}\left[u\left(x,t\right)-u\left(x,t_1\right)\right]$$
$$+\frac{1}{2}v\left(\frac{\partial}{\partial x}\left[\left(\frac{\partial u\left(x,t\right)}{\partial x}\right)^2-\left(\frac{\partial u\left(x,t_1\right)}{\partial x}\right)^2\right]\right)$$
$$+2\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial x}\left(\frac{\partial}{\partial x}\left[u\left(x,t\right)-u\left(x,t_1\right)\right]\right)-f_1\Big\|$$
$$\leq\left\|2\delta^2+\frac{1}{2}\lambda_2\delta\left(a+b\right)+\tau_2\left(2+\lambda_2\delta\right)+\xi_1\right\|$$
$$\left\|u\left(x,t\right)-u(x,t_1)\right\|$$
$$\leq\left(2\delta^2+\frac{1}{2}\lambda_2\delta\left(a+b\right)+\tau_2\left(2+\lambda_2\delta\right)+\xi_1\right)$$
$$\left\|u(x,t)-u(x,t_1)\right\|, \tag{37}$$

where $\|v\left(x,t\right)\| \leq \lambda_2$ be the bounded function, δ is the differential operator, $\left\|\frac{\partial v}{\partial x}\right\| \leq \tau_2$, $\left\|\frac{\partial u}{\partial x}\right\| \leq a$, $\left\|\frac{\partial u_1}{\partial x}\right\| \leq b$, and f_1 is also a bounded function ($\|f_1\| \leq \xi_1$). Putting $\eta_1 = 2\delta^2 + \lambda_2\delta\left(a+b\right) + \tau_2\left(2+\lambda_2\delta\right) + \xi_1$ in the above inequality, then we have

$$\|G_1(x,t,u)-G_1(x,t,u_1)\| \leq \eta_1\|u(x,t)-u(x,t_1)\|. \tag{38}$$

This gives that 0 the Lipschitz condition is obtained for G_2. Further, we can see that if $0 \leq \left(2\delta^2+\frac{1}{2}\lambda_2\delta\left(a+b\right)+\tau_2\left(2+\lambda_2\delta\right)+\xi_1\right) < 1$, then it implies the contraction. The remaining cases can be verified in a similar manner and which is given as follows:

$$\|G_2(x,t,v)-G_2(x,t,v_1)\| \leq \eta_2\|v(x,t)-v(x,t_1)\|. \tag{39}$$

The recursive form of Eq. (36) is defined as follows:

$$\begin{cases} u_n\left(x,\,t\right) = & \dfrac{(1-\alpha)}{B\left(\alpha\right)}G_1\left(x,t,u_{n-1}\right)+\dfrac{\alpha}{B\left(\alpha\right)\Gamma\left(\alpha\right)} \\ & \displaystyle\int_0^t G_1\left(x,\zeta,u_{n-1}\right)\left(t-\zeta\right)^{\alpha-1}d\zeta, \\ v_n\left(x,\,t\right) = & \dfrac{(1-\beta)}{B\left(\beta\right)}G_2\left(x,t,v_{n-1}\right)+\dfrac{\beta}{B\left(\beta\right)\Gamma\left(\beta\right)} \\ & \displaystyle\int_0^t G_2\left(x,\zeta,v_{n-1}\right)\left(t-\zeta\right)^{\beta-1}d\zeta. \end{cases} \tag{40}$$

The associated initial conditions are

$$u(x,0)=u_0(x,t) \text{ and } v(x,0)=v_0(x,t). \tag{41}$$

The successive difference between the terms is presented as follows:

$$\begin{cases} \phi_{1n}(x,t) = u_n(x,t)-u_{n-1}(x,t) \\ \qquad = \dfrac{(1-\alpha)}{B(\alpha)}(G_1\left(x,t,u_{n-1}\right) \\ \qquad\quad -G_1\left(x,t,u_{n-2}\right))+\dfrac{\alpha}{B(\alpha)\Gamma(\alpha)} \\ \qquad\quad \times\displaystyle\int_0^t G_1\left(x,\zeta,u_{n-1}\right)\left(t-\zeta\right)^{\alpha-1}d\zeta, \\ \phi_{2n}(x,t) = v_n(x,t)-v_{n-1}(x,t) \\ \qquad = \dfrac{(1-\beta)}{B(\beta)}(G_2\left(x,t,v_{n-1}\right) \\ \qquad\quad -G_2\left(x,t,v_{n-2}\right))+\dfrac{\beta}{B(\beta)\Gamma(\beta)} \\ \qquad\quad \times\displaystyle\int_0^t G_2\left(x,\zeta,v_{n-1}\right)\left(t-\zeta\right)^{\beta-1}d\zeta. \end{cases} \tag{42}$$

Note that

$$\begin{cases} u_n\left(x,t\right) = \displaystyle\sum_{i=1}^n \phi_{1i}\left(x,t\right), \\ v_n\left(x,t\right) = \displaystyle\sum_{i=1}^n \phi_{2i}\left(x,t\right). \end{cases} \tag{43}$$

By using Eq. (38) after applying the norm on the second equation of system (42), one can get

$$\begin{aligned} &\|\phi_{1n}(x,t)\| \\ &\quad\leq \frac{(1-\alpha)}{B(\alpha)}\eta_1\|\phi_{1(n-1)}(x,t)\| \\ &\qquad+\frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1\int_0^t \|\phi_{1(n-1)}(x,\zeta)\|d\zeta. \end{aligned} \tag{44}$$

Similarly, we have

$$\begin{aligned} &\|\phi_{2n}(x,t)\| \\ &\quad\leq \frac{(1-\beta)}{B(\beta)}\eta_2\|\phi_{2(n-1)}(x,t)\| \\ &\qquad+\frac{\beta}{B(\beta)\Gamma(\beta)}\eta_2\int_0^t \|\phi_{2(n-1)}(x,\zeta)\|d\zeta. \end{aligned} \tag{45}$$

□

We prove the following theorem by using the above result.

Theorem 4. *The solution for system* (27) *will exist and unique if we have specific* t_0 *then*

$$\frac{(1-\alpha)}{B(\alpha)}\eta_1 + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1 < 1,$$

$$\frac{(1-\beta)}{B(\beta)}\eta_2 + \frac{\beta}{B(\beta)\Gamma(\beta)}\eta_2 < 1.$$

Proof. Let us consider the bounded functions $u(x,t)$ and $v(x,t)$ satisfying the Lipschitz condition. Then, by Eqs. (43) and (45), we have

$$\|\phi_{1i}(x,t)\| \le \|u_n(x,0)\| \left[\frac{(1-\alpha)}{B(\alpha)}\eta_1 + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1\right]^n,$$
$$\|\phi_{2i}(x,t)\| \le \|v_n(x,0)\| \left[\frac{(1-\beta)}{B(\beta)}\eta_2 + \frac{\beta}{B(\beta)\Gamma(\beta)}\eta_2\right]^n. \tag{46}$$

Therefore, the continuity as well as existence for the obtained solutions is proved. Subsequently, in order to show that system (46) is a solution for system (27), we consider

$$\begin{aligned} u(x,t) - u(x,0) &= u_n(x,t) - K_{1n}(x,t), \\ v(x,t) - v(x,0) &= v_n(x,t) - K_{2n}(x,t). \end{aligned} \tag{47}$$

In order to obtain a result, we consider

$$\begin{aligned} &\|K_{1n}(x,t)\| \\ &\quad = \left\| \frac{(1-\alpha)}{B(\alpha)}(G_1(x,t,u) - G_1(x,t,\, u_{n-1})) \right. \\ &\qquad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (t-\zeta)^{\mu-1}\,(G_1(x,\zeta,u) \\ &\qquad -G_1(x,\zeta,u_{n-1}))d\zeta \le \frac{(1-\alpha)}{B(\alpha)}\|(G_1(x,t,u) \\ &\qquad -G_1(x,t,\, u_{n-1}))\| + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \\ &\qquad \times \int_0^t \|(G_1(x,\zeta,u) - G_1(x,\zeta,u_{n-1}))\| d\zeta \\ &\quad \le \frac{(1-\alpha)}{B(\alpha)}\eta_1 \|u - u_{n-1}\| \\ &\qquad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1 \|u - u_{n-1}\|\, t. \end{aligned} \tag{48}$$

Similarly, at t_0 we can obtain

$$\|K_{1n}(x,t)\| \le \left(\frac{(1-\alpha)}{B(\alpha)} + \frac{\alpha\, t_0}{B(\alpha)\Gamma(\alpha)}\right)^{n+1} \eta_1^{n+1} M. \tag{49}$$

As n approaches to ∞, we can see that from Eq. (50), $\|K_{1n}(x,t)\|$ tends to 0. Similarly, we can verify for $\|K_{2n}(x,t)\|$.

Next, it is a necessity to demonstrate uniqueness for the solution of the considered model. Suppose $u^*(x,t)$and $v^*(x,t)$ be the set of other solutions, then we have

$$\begin{aligned} u(x,t) - u^*(x,t) = \frac{(1-\alpha)}{B(\alpha)}&(G_1(x,t,u) \\ &-G_1(x,t,\, u^*)) \\ &+ \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (G_1(x,\zeta,u) \\ &-G_1(x,\zeta,u^*))d\zeta. \end{aligned} \tag{50}$$

On applying norm, Eq. (50) simplifies to

$$\begin{aligned} &\|u(x,t) - u^*(x,t)\| \\ &= \left\| \frac{(1-\alpha)}{B(\alpha)}(G_1(x,t,u) - G_1(x,t,\, u^*)) \right. \\ &\quad \left. + \frac{\alpha}{B(\alpha)\Gamma(\alpha)} \int_0^t (G_1(x,\zeta,u) - G_1(x,\zeta,u^*))d\zeta \right\| \\ &\le \frac{(1-\alpha)}{B(\alpha)}\eta_1 \|u(x,t) - u^*(x,t)\| \\ &\quad + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1 t \|u(x,t) - u^*(x,t)\|. \end{aligned} \tag{51}$$

On simplification

$$\begin{aligned} \|u(x,t) - u^*(x,t)\| &\left(1 - \frac{(1-\alpha)}{B(\alpha)}\eta_1 \right. \\ &\left. - \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1 t\right) \le 0. \end{aligned} \tag{52}$$

From the above condition, it is clear that $u(x,t) - u^*(x,t)$, if

$$\left(1 - \frac{(1-\alpha)}{B(\alpha)}\eta_1 - \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\eta_1 t\right) \ge 0. \tag{53}$$

□

Hence, Eq. (53) evidences our essential result.

Theorem 5. *Suppose* $u_n(x,t)$, $v_n(x,t)$, $u(x,t)$, *and* $v(x,t)$ *are defined in the Banach space* $(B[0,\,T], \|\cdot\|)$. *Then series solution defined in Eq.* (26) *converges to the solution of Eq.* (10), *if* $0 < \lambda_1 < 1$ *and* $0 < \lambda_2 < 1$.

Proof. Let us consider the sequence $\{S_n\}$ and which is the partial sum of Eq. (26), then we have to prove $\{S_n\}$ is Cauchy sequence in $(B[0,\,T], \|\cdot\|)$. Now consider

$$\begin{aligned} &\|S_{n+1}(x,t) - S_n(x,t)\| \\ &\quad = \|u_{n+1}(x,t)\| \le \lambda_1 \|u_n(x,t)\| \\ &\quad \le \lambda_1^2 \|u_{n-1}(x,t)\| \le \cdots \le \lambda_1^{n+1} \|u_0(x,t)\|. \end{aligned} \tag{54}$$

Now, we have for every $n,\, m \in N\, (m \leq n)$

$$\begin{aligned}\|S_n - S_m\| &= \|(S_n - S_{n-1}) + (S_{n-1} - S_{n-2}) \\ &\quad + \cdots + (S_{m+1} - S_m)\| \\ &\leq \|S_n - S_{n-1}\| + \|S_{n-1} - S_{n-2}\| + \cdots \\ &\quad + \|S_{m+1} - S_m\| \\ &\leq \left(\lambda_1^n + \lambda_1^{n-1} + \cdots + \lambda_1^{m+1}\right)\|u_0\| \\ &\leq \lambda_1^{m+1}(\lambda_1^{n-m-1} + \lambda_1^{n-m-2} + \cdots \\ &\quad + \lambda_1 + 1)\|u_0\| \\ &\leq \lambda_1^{m+1}\left(\frac{1-\lambda_1^{n-m}}{1-\lambda_1}\right)\|u_0\|. \end{aligned} \tag{55}$$

But $0 < \lambda_1 < 1$, therefore $\|S_n - S_m\| = 0$. Hence, $\{S_n\}$ is the Cauchy sequence. Similarly, we can demonstrate for the second case. This proves the required result. □

Theorem 6. *The maximum absolute error of the series solution* (26) *of Eq.* (10) *is estimated as*

$$\left\|u(x,t) - \sum_{n=0}^{M} u_n(x,t)\right\| \leq \frac{\lambda_1^{M+1}}{1-\lambda_1}\|u_0(x,t)\|.$$

Proof. By the help of Eq. (55), we get

$$\|u(x,t) - S_n\| = \lambda_1^{m+1}\left(\frac{1-\lambda_1^{n-m}}{1-\lambda_1}\right)\|u_0(x,t)\|.$$

But $0 < \lambda_1 < 0 \Rightarrow 1 - \lambda_1^{n-m} < 1$. Hence, we have

$$\left\|u(x,t) - \sum_{n=0}^{M} u_n(x,t)\right\| \leq \frac{\lambda_1^{M+1}}{1-\lambda_1}\|u_0(x,t)\|.$$

This ends the proof. □

6. NUMERICAL RESULTS AND DISCUSSION

Here, we consider three different cases in order to present applicability of the future scheme with distinct initial conditions.

Case 1. Consider Eqs. (27) and (28) as

$$\begin{aligned} &{}_a^{ABC}D_t^{\alpha+1}u(x,t) - (2 - vu_x)u_{xx} \\ &\quad + (2 + vu_x)v_x - f_1(x,t) = 0, \\ &{}_a^{ABC}D_t^{\beta}v(x,t) + (2 + vu_x)u_{xt} \\ &\quad - vv_{xx} - f_2(x,t) = 0, \end{aligned} \tag{56}$$

with

$$\begin{aligned} u(x,0) = v(x,0) &= \frac{1}{1+x^2}, \\ u_t(x,0) &= 0. \end{aligned} \tag{57}$$

The analytical solution for the proposed system is

$$u(x,t) = \frac{1+t^2}{1+x^2}, \quad v(x,t) = \frac{1+t}{1+x^2}. \tag{58}$$

Now, consider

$$\begin{aligned} f_1(x,t) &= \frac{2}{1+x^2} - \frac{2\left(1+t^2\right)\left(3x^2-1\right)}{\left(1+x^2\right)^3}a(w_1,\, w_2) \\ &\quad - \frac{2x(1+t)}{\left(1+x^2\right)^2}b(w_1,\, w_2), \\ f_2(x,t) &= \frac{1}{1+x^2}c(w_1, w_2) - \frac{4xt}{\left(1+x^2\right)^2}b(w_1,\, w_2) \\ &\quad - \frac{2\left(3x^2-1\right)(1+t)}{\left(1+x^2\right)^3}d(w_2), \\ w_1(x,t) &= -\frac{2x\left(1+t^2\right)}{\left(1+x^2\right)^2} \end{aligned}$$

and

$$w_2(x,t) = \frac{1+t}{1+x^2}.$$

Then, we can obtain the terms of the series solution by using the initial conditions

$$u_0(x,t) = \frac{1}{1+x^2} \text{ and } v_0(x,t) = \frac{1}{1+x^2}.$$

Case 2. Consider fractional nonlinear coupled system describing thermoelasticity of the form:

$$\begin{aligned} &{}_a^{ABC}D_t^{\alpha+1}u(x,t) - u_{xx} + (vu_x)v_x \\ &\quad + e^{-x+t} = 0,\ 0 < \alpha \leq 1, \\ &{}_a^{ABC}D_t^{\beta}v(x,t) - v_{xx} + (vu_x)u_{xt} \\ &\quad + e^{x-t} = 0, \quad 0 < \beta \leq 1, \end{aligned} \tag{59}$$

with

$$\begin{aligned} u(x,0) = e^x,\ v(x,0) &= e^{-x}, \\ u_t(x,0) &= -e^x. \end{aligned} \tag{60}$$

The analytical solution for the proposed system is

$$u(x,t) = e^{x-t}, \quad v(x,t) = e^{-x+t}. \tag{61}$$

Then, we can obtain the terms of the series solution using $u_0(x,t) = e^x(1-t)$ and $v_0(x,t) = e^{-x}$.

Case 3. Consider fractional nonlinear coupled system describing thermoelasticity of the form

$$\begin{aligned} &{}_a^{ABC}D_t^{\alpha+1}u(x,t) - (vu_x)_x + v_x - 2x + 6x^2 \\ &\quad + 2t^2 + 2 = 0, \quad 0 < \alpha \leq 1, \\ &{}_a^{ABC}D_t^{\beta}v(x,t) - (uv_x)_x + u_{xt} - 2t^2 - 2t \\ &\quad + 6x^2 = 0,\ 0 < \beta \leq 1, \end{aligned} \tag{62}$$

Table 1 **Comparison of q-HATM Solution with VIM[18] at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$ for Different x and t.**

x	$\lvert u_{\text{Exact}} - u_{\text{VIM}} \rvert$	$\lvert u_{\text{Exact}} - u_{q\text{-HATM}} \rvert$	$\lvert v_{\text{Exact}} - v_{\text{VIM}} \rvert$	$\lvert v_{\text{Exact}} - v_{q\text{-HATM}} \rvert$
5	1.14214×10^{-5}	1.90096×10^{-6}	1.01307×10^{-4}	5.98056×10^{-6}
6	5.66626×10^{-6}	1.12189×10^{-6}	4.88405×10^{-5}	3.52874×10^{-6}
7	3.11183×10^{-6}	7.14536×10^{-7}	2.64856×10^{-5}	2.24926×10^{-6}
8	1.84483×10^{-6}	4.82043×10^{-7}	1.55626×10^{-5}	1.51905×10^{-6}
9	1.16070×10^{-6}	3.40099×10^{-7}	9.72698×10^{-6}	1.07293×10^{-6}
1	07.65770×10^{-7}	2.48697×10^{-7}	6.38473×10^{-6}	7.85384×10^{-7}
1	15.25173×10^{-7}	1.87252×10^{-7}	4.36097×10^{-6}	5.91892×10^{-7}
1	23.71954×10^{-7}	1.44453×10^{-7}	3.07843×10^{-6}	4.56989×10^{-7}
1	32.70692×10^{-7}	1.13744×10^{-7}	2.23417×10^{-6}	3.60104×10^{-7}
1	42.01627×10^{-7}	9.11447×10^{-8}	1.66026×10^{-6}	2.88749×10^{-7}
1	51.53229×10^{-7}	7.41497×10^{-8}	1.25921×10^{-6}	2.35048×10^{-7}

Table 2 **Numerical Stimulation for $u(x,t)$ of q-HATM Solution at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$ with Different x and t.**

x	t	$\lvert u_{\text{Exact}} - u^{(2)} \rvert$	$\lvert u_{\text{Exact}} - u^{(3)} \rvert$	$\lvert u_{\text{Exact}} - u^{(4)} \rvert$
0.25	0.025	1.67755×10^{-3}	2.33885×10^{-4}	1.20577×10^{-6}
	0.05	6.92276×10^{-3}	9.54352×10^{-4}	1.64025×10^{-5}
	0.075	1.59986×10^{-2}	2.21470×10^{-3}	7.85886×10^{-5}
	0.1	2.89892×10^{-2}	4.09961×10^{-3}	2.42402×10^{-4}
0.50	0.025	5.05158×10^{-4}	6.40525×10^{-5}	3.57226×10^{-6}
	0.05	2.18546×10^{-3}	2.49191×10^{-4}	2.62338×10^{-5}
	0.075	5.35146×10^{-3}	5.58497×10^{-4}	8.03665×10^{-5}
	0.1	1.04013×10^{-2}	1.01489×10^{-3}	1.70485×10^{-4}
0.75	0.025	2.05386×10^{-4}	3.66385×10^{-5}	7.03323×10^{-6}
	0.05	7.97441×10^{-4}	1.74416×10^{-4}	5.70561×10^{-5}
	0.075	1.68825×10^{-3}	4.54420×10^{-4}	1.95176×10^{-4}
	0.1	2.68415×10^{-3}	9.15173×10^{-4}	4.68693×10^{-4}
1	0.025	3.75300×10^{-4}	5.92073×10^{-5}	7.03125×10^{-6}
	0.05	1.54577×10^{-3}	2.67576×10^{-4}	5.78125×10^{-5}
	0.075	3.57446×10^{-3}	6.76075×10^{-4}	2.00391×10^{-4}
	0.1	6.50367×10^{-3}	1.34166×10^{-3}	4.87506×10^{-4}

with

$$\begin{aligned} u(x,0) = v(x,0) = x^2, \\ u_t(x,0) = 0. \end{aligned} \tag{63}$$

The analytical solution for the proposed system is

$$u(x,t) = x^2 - t^2, \quad v(x,t) = x^2 + t^2. \tag{64}$$

Then, we can obtain the terms of the series solution with initial conditions

$$u_0(x,t) = x^2 \text{ and } v_0(x,t) = x^2.$$

In the present investigation, we find the solution for coupled equations arising in thermoelasticity having arbitrary order using a novel scheme, namely, q-HATM with the help of Mittag-Leffler law. In the present segment, we demonstrate the numerical simulation for the considered coupled system considered in Case 1, which is cited in Tables 1–3. Table 1 particularly shows the comparison of obtained solution with solution obtained by VIM in terms of absolute error. Further, in Tables 2 and 3, we demonstrated the efficiency of the future method and we conform that as number of iterations increases the obtained solution gets close to analytical solution. From the tables, we can see that the proposed scheme is more accurate.

On the contrary, in order to capture the behavior of q-HATM solution for diverse value of the parameters we plot the 2D and 3D plots. In Fig. 1,

Table 3 Numerical Stimulation for $v(x,t)$ of q-HATM Solution at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$ with Different x and t.

x	t	$\left\lvert v_{\text{Exact}} - v^{(2)} \right\rvert$	$\left\lvert v_{\text{Exact}} - v^{(3)} \right\rvert$	$\left\lvert v_{\text{Exact}} - v^{(4)} \right\rvert$
0.25	0.025	1.09084×10^{-1}	5.64302×10^{-2}	7.43209×10^{-4}
	0.05	2.13480×10^{-1}	1.15295×10^{-1}	3.07744×10^{-3}
	0.075	3.08307×10^{-1}	1.77200×10^{-1}	7.16258×10^{-3}
	0.1	3.84297×10^{-1}	2.42780×10^{-1}	1.31628×10^{-2}
0.50	0.025	5.55727×10^{-2}	2.37751×10^{-2}	9.20698×10^{-4}
	0.05	1.22561×10^{-1}	4.59247×10^{-2}	3.62578×10^{-3}
	0.075	2.00233×10^{-1}	6.81455×10^{-2}	8.02443×10^{-3}
	0.1	2.85014×10^{-1}	9.22703×10^{-2}	1.40182×10^{-2}
0.75	0.025	2.01840×10^{-2}	7.84490×10^{-3}	1.51823×10^{-3}
	0.05	4.10058×10^{-2}	1.13435×10^{-2}	6.05474×10^{-3}
	0.075	6.49530×10^{-2}	1.11965×10^{-2}	1.35784×10^{-2}
	0.1	9.54470×10^{-2}	8.18341×10^{-3}	2.40522×10^{-2}
1	0.025	9.47747×10^{-3}	3.95993×10^{-3}	1.40615×10^{-3}
	0.05	1.31780×10^{-2}	3.35122×10^{-3}	5.62336×10^{-3}
	0.075	1.18853×10^{-2}	1.79649×10^{-3}	1.26477×10^{-2}
	0.1	6.94542×10^{-3}	1.14357×10^{-2}	2.24725×10^{-2}

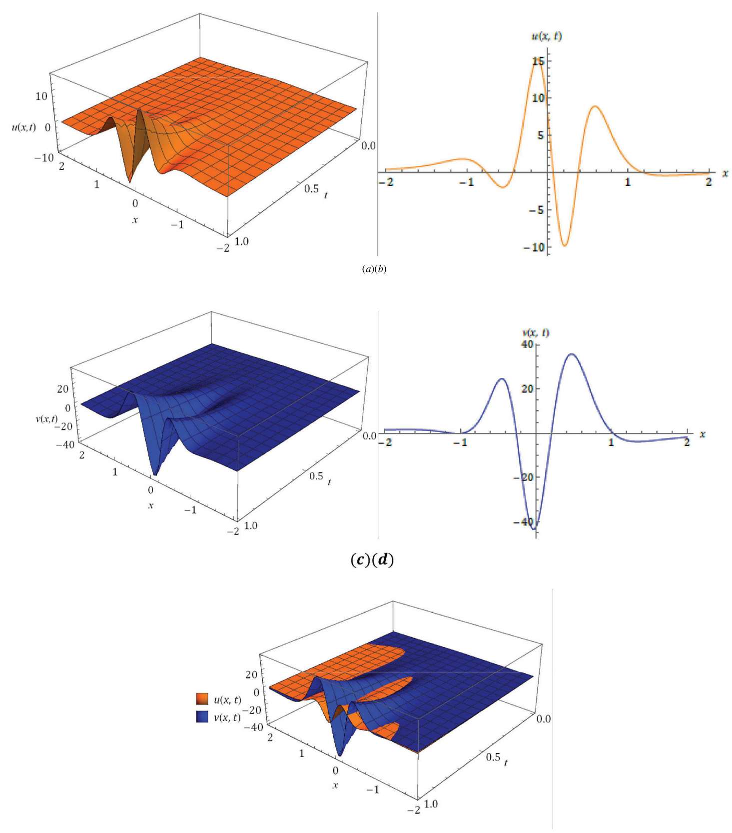


Fig. 1 **(a)** Surface of $u(x,t)$, **(b)** 2D plot of $u(x,t)$ at $t = 1$, **(c)** surface of $v(x,t)$, **(d)** 2D plot of $v(x,t)$ at $t = 1$, **(e)** coupled surface of the obtained solution cited in Case 1 at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$.

we present the nature of q-HATM solution and coupled surface of the obtained solution for coupled system defined in Case 1. The coupled surface of the obtained solution for the proposed model has been illustrated in order to understand the physical behavior of the coupled system. The natures of q-HATM solution for different arbitrary orders are presented in Fig. 2 in terms of 2D plots. Similarly, we capture the physical variation of considered coupled system defined in Case 2 and Case 3 and are, respectively, presented in Figs. 4 and 7 in terms of 3D plots with coupled surfaces at classical order. Meanwhile, the response of q-HATM solution for different arbitrary orders has been demonstrated in Figs. 5 and 8 for Case 2 and Case 3, respectively. In order to analyze the behavior of obtained solution with respect to homotopy parameter($\hbar$), the $\hbar$-curves are drowned with diverse μ and presented in Figs. 3, 6, and 9 for Cases 1–3. These curves aid to control and adjust the convergence region of the q-HATM solution. Meanwhile, the horizontal line in the plots represents the convergence region. For an appropriate value of $\hbar$, the obtained solution quickly converges to exact solution. These plots aid us to simulate and exhibit the physical properties of nonlinear phenomena arising in science and technology in order to study and analyze their nature with the aid of FC. Moreover, from all

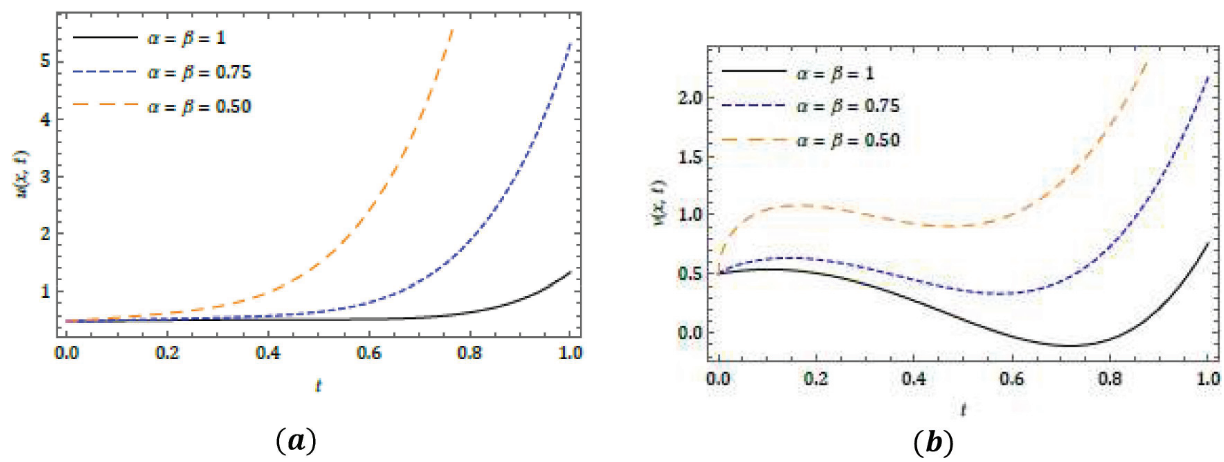


Fig. 2 Nature of the q-HATM solution defined in Case 1 for **(a)** $u(x, t)$ and **(b)** $v(x, t)$ with distinct α and β at $\hbar = -1$, $n = 1$, and $x = 1$.

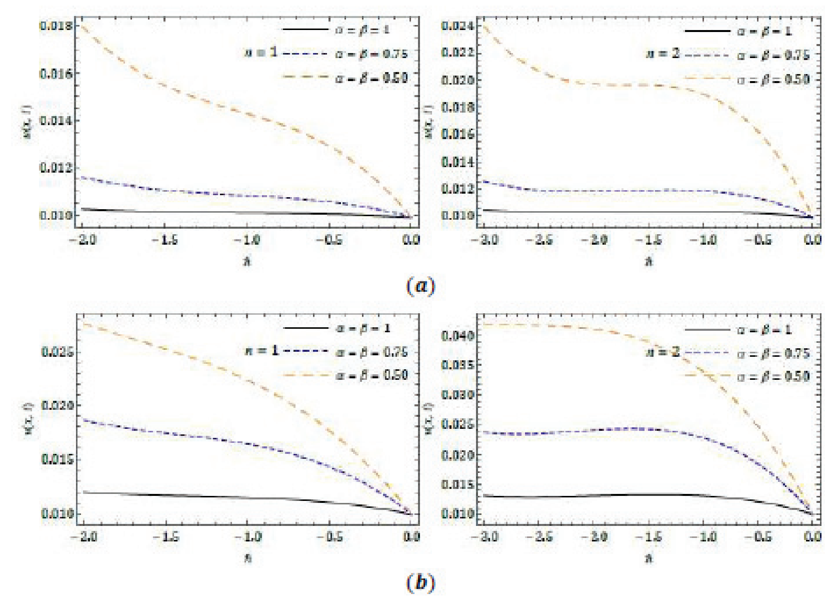
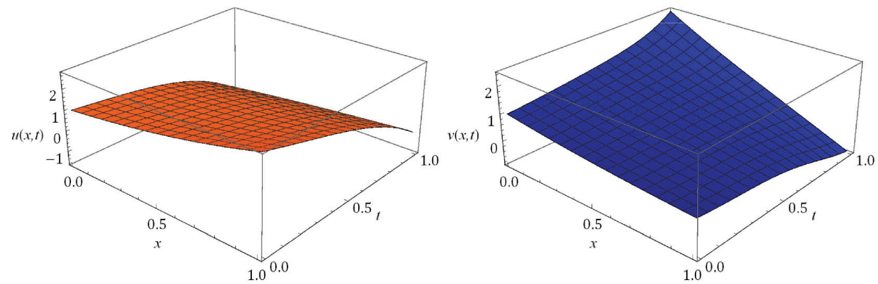
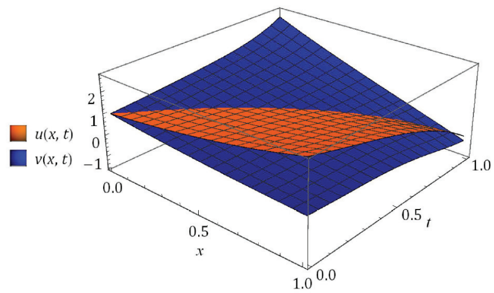


Fig. 3 $\hbar$-Curves q-HATM solution cited in Case 1 for **(a)** $u(x, t)$ and **(b)** $v(x, t)$ with distinct α and β at $x = 1$, $t = 0.01$ for $n = 1$ and 2.

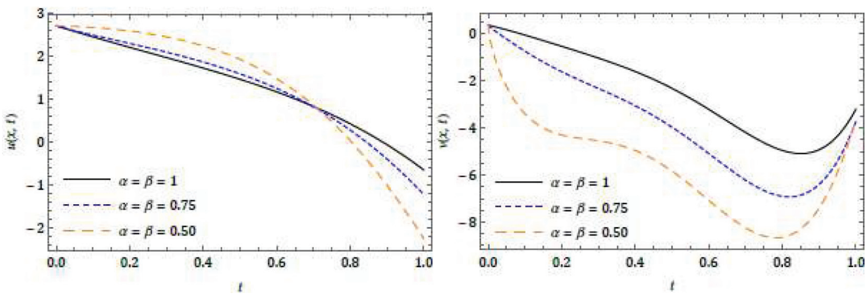


$(\boldsymbol{a})(\boldsymbol{b})$



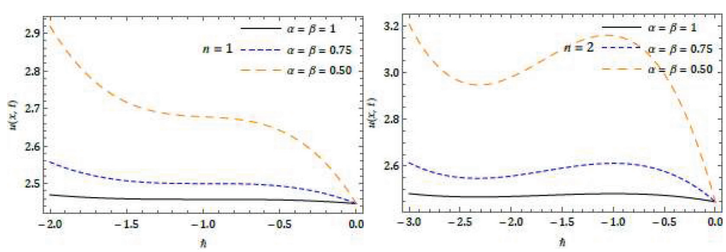
$(\boldsymbol{c})$

Fig. 4 Surfaces of **(a)** $u\,(x,t)$, **(b)** $v\,(x,t)$ and **(c)** coupled surface of the obtained solution cited in Case 2 at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$.

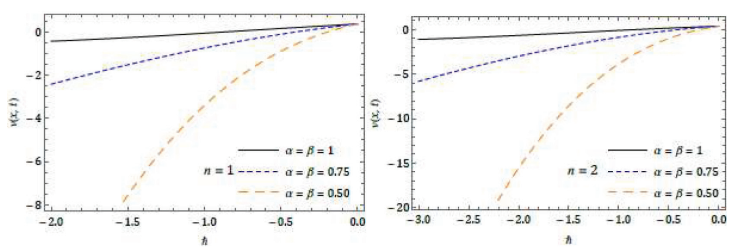


$(\boldsymbol{a})(\boldsymbol{b})$

Fig. 5 Nature of the q-HATM solution defined in Case 2 for **(a)** $u\,(x,\,t)$ and **(b)** $v\,(x,t)$ with distinct α and β at $\hbar = -1$, $n = 1$, and $x = 1$.

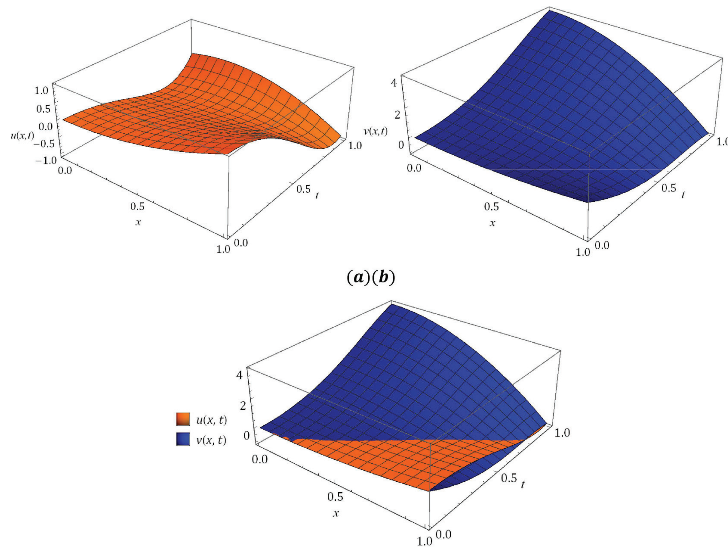


$(\boldsymbol{a})$



$(\boldsymbol{b})$

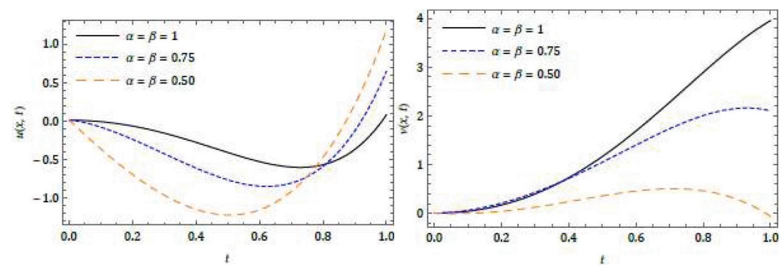
Fig. 6 $\hbar$-curves for achieved solution considered in Case 2 of **(a)** $u\,(x,\,t)$ and **(b)** $v\,(x,t)$ with distinct α and β at $x = 0.1$, $t = 0.1$ for $n = 1$ and 2.



(a)(b)

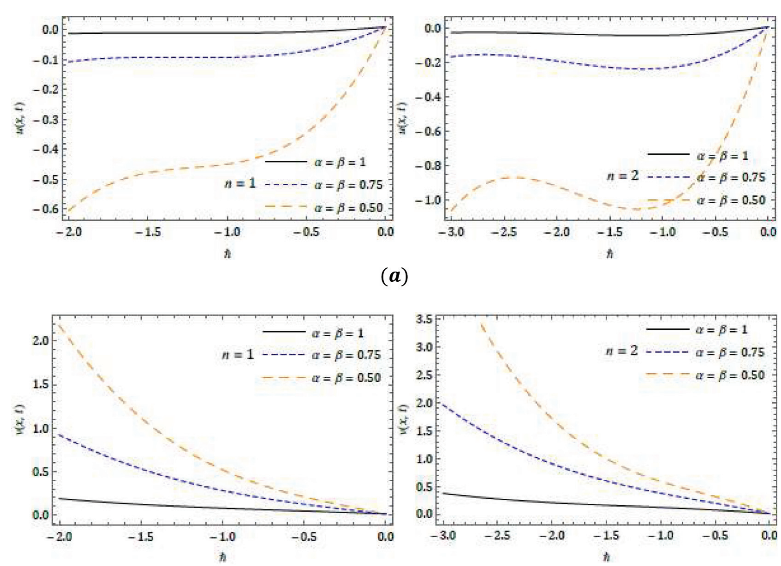
(c)

Fig. 7 Surfaces of **(a)** $u(x,t)$, **(b)** $v(x,t)$, and **(c)** coupled surface of the obtained solution cited in Case 3 at $\hbar = -1$, $n = 1$, and $\alpha = \beta = 1$.



(a)(b)

Fig. 8 Nature of the q-HATM solution defined in Case 3 for **(a)** $u(x,\ t)$ and **(b)** $v(x,\ t)$ with distinct α and β at $\hbar = -1$, $n = 1$, and $x = 0.1$.



(a)

(b)

Fig. 9 $\hbar$-Curves q-HATM solution cited in Case 3 for **(a)** $u(x,\ t)$ and **(b)** $v(x,\ t)$ with distinct α and β at $x = 0.1$, $t = 0.1$ for $n = 1$ and 2.

the plots, we can see that the proposed method is more accurate and very effective to analyze the considered complex coupled fractional order equations.

7. CONCLUSION

In this study, the q-HATM is applied lucratively to find the solution for fractional coupled systems arising in thermoelasticity. Since AB derivatives and integrals having fractional order are defined with the help of generalized Mittag-Leffler function as the nonsingular and nonlocal kernel, the present investigation illuminates the effectiveness of the considered derivative. The existence and uniqueness of the obtained solution are demonstrated with the fixed point hypothesis. The results obtained by the future scheme are more stimulating when compared with results available in the literature. Further, the proposed algorithm finds the solution of the coupled nonlinear problem without considering any discretization, perturbation, or transformations. The present investigation illuminates the considered nonlinear phenomena, which noticeably depend on the time history and the time instant and which can be proficiently analyzed by applying the concept of calculus with fractional order. The present investigation helps the researchers to study the behavior nonlinear problems, which give very interesting and useful consequences. Lastly, we can conclude that the projected method is extremely methodical, more effective, and very accurate, and which can be applied to analyze the diverse classes of coupled nonlinear problems.

REFERENCES

1. J. Liouville, Memoire surquelques questions de geometrieet de mecanique, etsurun nouveau genre de calcul pour resoudreces questions, *J. Ecole. Polytech.* **13** (1832) 1–69.
2. G. F. B. Riemann, Versuch Einer Allgemeinen Auffassung der Integration und Differentiation, Gesammelte Mathematische Werke, Leipzig (1896).
3. M. Caputo, Elasticita e Dissipazione, Zanichelli, Bologna (1969).
4. K. S. Miller and B. Ross, *An Introduction to Fractional Calculus and Fractional Differential Equations* (A Wiley, New York, 1993).
5. I. Podlubny, *Fractional Differential Equations* (Academic Press, New York, 1999).
6. A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations* (Elsevier, Amsterdam, 2006).
7. D. Baleanu, Z. B. Guvenc and J. A. Tenreiro Machado, *New Trends in Nanotechnology and Fractional Calculus Applications* (Springer, Heidelberg, 2010).
8. A. Esen, T. A. Sulaiman, H. Bulut and H. M. Baskonus, Optical solitons and other solutions to the conformable space-time fractional Fokas–Lenells equation, *Optik* **167** (2018) 150–156.
9. P. Veeresha, D. G. Prakasha and H. M. Baskonus, Solving smoking epidemic model of fractional order using a modified homotopy analysis transform method, *Math. Sci.* **13**(2) (2019) 115–128.
10. D. Baleanu, G. C. Wu and S. D. Zeng, Chaos analysis and asymptotic stability of generalized Caputo fractional differential equations, *Chaos Solitons Fractals* **102** (2017) 99–105.
11. P. Veeresha, D. G. Prakasha and H. M. Baskonus, New numerical surfaces to the mathematical model of cancer chemotherapy effect in Caputo fractional derivatives, *Chaos* **29** (2019), doi:10.1063/1.5074099.
12. H. M. Baskonus, T. A. Sulaiman and H. Bulut, On the new wave behavior to the Klein–Gordon–Zakharov equations in plasma physics, *Ind. J. Phys.* **93**(3) (2019) 393–399.
13. D. G. Prakasha, P. Veeresha and H. M. Baskonus, Analysis of the dynamics of hepatitis E virus using the Atangana–Baleanu fractional derivative, *Eur. Phys. J. Plus* **134**(241) (2019) 1–11, https://doi.org/10.1140/epjp/i2019-12590-5.
14. P. Veeresha, D. G. Prakasha and D. Baleanu, An efficient numerical technique for the nonlinear fractional Kolmogorov–Petrovskii–Piskunov equation, *Mathematics* **7**(3) (2019) 1–18, doi:10.3390/math7030265.
15. D. E. Carlson, Linear thermoelasticity, *Linear Theor. Elast. Thermoelast.* (1972) 297–345, doi:10.1007/978-3-662-39776-3_2.
16. O. W. Dillon, A nonlinear thermoelasticity theory, *J. Mech. Phys. Solids* **10** (1962) 123–131.
17. S. Jiang, Numerical solution for the Cauchy problem in nonlinear 1-D-thermoelasticity, *Computing* **44** (1990) 147–158.
18. N. H. Sweilam and M. M. Khader, Variational iteration method for one dimensional nonlinear thermoelasticity, *Chaos Solitons Fractals* **32** (2007) 145–149.
19. M. Slemrod, Global existence, uniqueness and asymptotic stability of classical solutions in one dimensional nonlinear thermoelasticity, *Arch. Ration. Mech. Anal.* **76** (1981) 97–133.
20. H. E. Gadain, Coupled singular and non-singular thermoelastic system and double Laplace decomposition method, *New Trends Math. Sci.* **4**(3) (2016) 212–222.
21. D. C. A Moura, *A Linear Uncoupling Numerical Scheme for the Nonlinear Coupled Thermodynamics*

Equations, Lecture Notes in Mathematics (Springer, Berlin, 1983).
22. A. Sadighi and D. D. Ganji, A study on one dimensional nonlinear thermoelasticity by Adomian decomposition method, *World J. Model. Simul.* **4** (2008) 19–25.
23. M. Caputo and M. Fabrizio, A new definition of fractional derivative without singular kernel, *Progr. Fract. Diff. Appl.* **1**(2) (2015) 73–85.
24. A. Atangana and D. Baleanu, New fractional derivatives with non-local and non-singular kernel theory and application to heat transfer model, *Therm. Sci.* **20** (2016) 763–769.
25. S. J. Liao, Homotopy analysis method and its applications in mathematics, *J. Basic Sci. Eng.* **5**(2) (1997) 111–125.
26. S. J. Liao, Homotopy analysis method: A new analytic method for nonlinear problems, *Appl. Math. Mech.* **19** (1998) 957–962.
27. J. Singh, D. Kumar and R. Swroop, Numerical solution of time- and space-fractional coupled Burgers' equations via homotopy algorithm, *Alex. Eng. J.* **55**(2) (2016) 1753–1763.
28. H. M. Srivastava, D. Kumar and J. Singh, An efficient analytical technique for fractional model of vibration equation, *Appl. Math. Model.* **45** (2017) 192–204.
29. D. G. Prakasha, P. Veeresha and H. M. Baskonus, Two novel computational techniques for fractional Gardner and Cahn–Hilliard equations, *Comput. Math. Methods* **1**(2) (2019), https://doi.org/10.1002/cmm4.1021.
30. H. Bulut, D. Kumar, J. Singh, R. Swroop and H. M. Baskonus, Analytic study for a fractional model of HIV infection of CD4+T lymphocyte cells, *Math. Nat. Sci.* **2**(1) (2018) 33–43.
31. K. S. Al-Ghafri and H. Rezazadeh, Solitons and other solutions of (3 + 1)-dimensional space–time fractional modified KdV–Zakharov–Kuznetsov equation, *Appl. Math. Nonlinear Sci.* **4**(2) (2019) 289–304.
32. P. Veeresha and D. G. Prakasha, Solution for fractional Zakharov–Kuznetsov equations by using two reliable techniques, *Chin. J. Phys.* **60** (2019) 313–330.
33. A. Cordero, J. P. Jaiswal and J. R. Torregrosa, Stability analysis of fourth-order iterative method for finding multiple roots of non-linear equations, *Appl. Math. Nonlinear Sci.* **4**(1) (2019) 43–56.
34. A. Prakash, P. Veeresha, D. G. Prakasha and M. Goyal, A homotopy technique for fractional order multi-dimensional telegraph equation via Laplace transform, *Eur. Phys. J. Plus* **134**(19) (2019) 1–18, doi:10.1140/epjp/i2019-12411-y.
35. A. Atangana and B. T. Alkahtani, Analysis of the Keller–Segel model with a fractional derivative without singular kernel, *Entropy* **17** (2015) 4439–4453.
36. A. Atangana and B. T. Alkahtani, Analysis of non-homogenous heat model with new trend of derivative with fractional order, *Chaos Solitons Fractals* **89** (2016) 566–571.
37. D. G. Prakasha, P. Veeresha and H. M. Baskonus, Analysis of the dynamics of hepatitis E virus using the Atangana–Baleanu fractional derivative, *Eur. Phys. J. Plus* **134**(241) (2019) 1–11, doi:10.1140/epjp/i2019-12590-5.
38. A. Atangana and D. Baleanu, New fractional derivatives with non-local and non-singular kernel theory and application to heat transfer model, *Therm. Sci.* **20** (2016) 763–769.
39. A. Atangana, Non validity of index law in fractional calculus: A fractional differential operator with Markovian and non-Markovian properties, *Physica A* **505** (2018) 688–706.
40. W. Gao, B. Ghanbari and H. M. Baskonus, New numerical simulations for some real world problems with Atangana–Baleanu fractional derivative, *Chaos Solitons Fractals* **128** (2019) 34–43.
41. J. Singh, D. Kumar, Z. Hammouch and A. Atangana, A fractional epidemiological model for computer viruses pertaining to a new fractional derivative, *Appl. Math. Comput.* **316** (2018) 504–515.
42. A. Atangana, On the stability and convergence of the time-fractional variable order telegraph equation, *J. Comput. Phys.* **293** (2015) 104–114.
43. A. Yokus and S. Gulbahar, Numerical solutions with linearization techniques of the fractional Harry Dym equation, *Appl. Math. Nonlinear Sci.* **4**(1) (2019) 35–42.
44. A. Prakash, D. G. Prakasha and P. Veeresha, A reliable algorithm for time-fractional Navier–Stokes equations via Laplace transform, *Nonlinear Eng.* (2019), doi:10.1515/nleng-2018-0080.
45. D. W. Brzeziński, Review of numerical methods for NumILPT with computational accuracy assessment for fractional calculus, *Appl. Math. Nonlinear Sci.* **3**(2) (2018) 487–502.
46. P. Veeresha, D. G. Prakasha and H. M. Baskonus, Novel simulations to the time- fractional Fisher's equation, *Math. Sci.* **13**(1) (2019) 33–42, doi: 10.1007/s40096-019-0276-6.
47. C. M. Khalique, O. D. Adeyemo and I. Simbanefayi, On optimal system, exact solutions and conservation laws of the modified equal-width equation, *Appl. Math. Nonlinear Sci.* **3**(2) (2018) 409–418.
48. D. Kumar, J. Singh, K. Tanwar and D. Baleanu, A new fractional exothermic reactions model having constant heat source in porous media with power, exponential and Mittag-Leffler laws, *Int. J. Heat Mass Transf.* **138** (2019) 1222–1227.