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On C_m-Supermagicness of Book-Snake Graphs

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Abstract.: Graph labelling or valuation is a function that maps graph elements (vertices or edges) to numbers, usually positive integers [7]. In this work, we analyze the supermagicness of book-snake graphs. At first, we show that triangular book-snake graph $S(3, n, s)$ admits a C_3-supermagic labelling. Then by using these labellings and supermagicness of subdivision given in [10], C_m-supermagicness of polygonal book-snake graph $S(m, n, s)$ is given since $S(m, n, s)$ can be taught as a subdivision of $S\,(3, n, s)$.

AMS (MOS) Subject Classification Codes: 05C78
Key Words: Magic Labelling, Covering, Book Graph, Snake Graph.

1. Introduction

A graph $G(V, E)$ is said to admit an $H-$covering if every edge is a member of a subgraph of G isomorphic to a simple graph H. A bijection $\lambda : V \cup E \to \{1, 2, \cdots, |V|+|E|\}$ is called an $H-$magic labelling of G if there exists an integer $m(\lambda)$ (called the magic sum) such that for any subgraph $H' = (V', E')$ of G isomorphic to H, $\sum\limits_{v\in V'} (\lambda(v)) + \sum\limits_{e\in E'} (\lambda(e)) = m(\lambda)$. A graph is said to be H magic if it satisfies the property of H magic labelling. If the $H-$magic labelling λ has the property such that $\{\lambda(v) : v \in V\} = \{1, 2, \cdots, |v|\}$, then λ is said to be $H-$supermagic labelling [2].

Several researchers studied $H-$supermagic labelling. For example:

$H-$supermagicness of Jahangir graphs, wheel graphs for even n, and complete bipartite graphs $K_{m,n}$ for $m = 2$ studied by Roswitha et al. [8]. Path-supermagic labelling was investigated by Maryati et al. [6]. Jeyanthi and Selvagopal [3] gave some formulations for supermagic labelling of book graphs by using equipartition. Supermagicness of generalized book is given by Kathiresan et al. [4]. Kojima [5] studied C_4-supermagic labelling on the cartesian product of paths and graphs. C_m-supermagic labelling on polygonal snake graphs was given by Selvagopal et al. [9]. More results can be found in [1].

In this paper, we define $m-$polygonal-book-snake graphs and show that they admit a C_m-supermagic labelling.

2. RESULTS

2.1. C_3-supermagicness of $S\,(3,n,s)$. The graph with $ns+s+1$ vertex and $2ns+s$ edges, obtained by joining s copies of the triangular book graph with $n-$leaves $B(3,n)$ along the common vertices, is called triangular-book-snake graph with the $n-$leaves and $s-$parts, and denoted by $S\,(3,n,s)$ (see Figure 1).

$$\begin{aligned} V =& \{v_i : i=1,2,\ldots,s+1\} \cup \{v_{ij} : i=1,2,\ldots,s, j=1,2,\ldots,n\}, \\ E =& \{e_i : i=1,2,\ldots,s\} \cup \{e^1_{ij} : i=1,2,\ldots,s, j=1,2,\ldots,n\} \cup \\ & \{e^2_{ij} : i=1,2,\ldots,s, j=1,2,\ldots,n\}. \end{aligned}$$

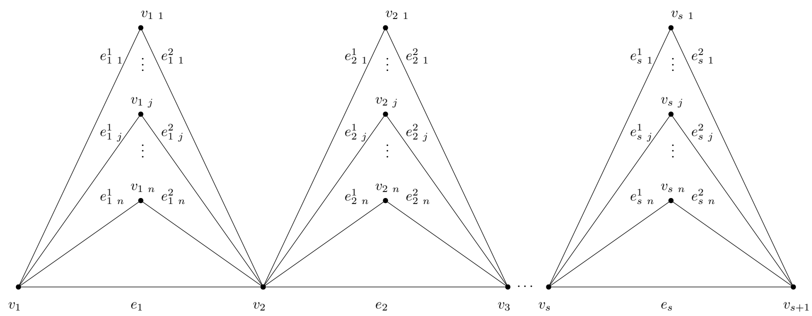

FIGURE 1. $S(3,n,s)$ graph

Theorem 2.2. *Let $n,s \geq 2$ and s even. Then $S(3,n,s)$ is C_3-supermagic.*

Proof. Let's define λ as follows:

$$\begin{aligned} \lambda(v_i) &= s+2-i, i=1,2,\ldots,s+1, \\ \lambda(v_{ij}) &= s+1+i+s(j-1), i=1,2,\ldots,s, j=1,2,\ldots,n, \\ \lambda(e_i) &= 3ns+s+i+1, i=1,2,\ldots,s, \\ \lambda(e^1_{ij}) &= \begin{cases} 2ns+\frac{3s}{2}-\frac{sj}{2}-i+2, & i=1,2,\ldots,\frac{s}{2}, j=1,2,...,n, \\ \frac{3ns}{2}+2s-\frac{sj}{2}-i+2, & i=\frac{s}{2}+1,\ldots,s, j=1,2,...,n, \end{cases} \\ \lambda(e^2_{ij}) &= \begin{cases} 2ns+s+s\frac{n-j}{2}+i+1, & i=1,2,\ldots,\frac{s}{2}, j=1,2,...,n, \\ 3ns+\frac{s}{2}-\frac{sj}{2}+i+1, & i=\frac{s}{2}+1,\ldots,s, j=1,2,...,n. \end{cases} \end{aligned}$$

Since we have

$$\sum_{v\in V'}\lambda(v) = v_i + v_{i+1} + v_{ij} = 3s + 4 + s(j-1) - i$$

and

$$\sum_{e\in E'}\lambda(e) = e_i + e^1_{ij} + e^2_{ij} = \frac{7}{2}s - js + \frac{15}{2}ns + 4 + i,$$

the supermagic sum $m(\lambda)$ is

$$m(\lambda) = \sum_{v\in V'}\lambda(v) + \sum_{e\in E'}\lambda(e) = \frac{11}{2}s + \frac{15}{2}ns + 8.$$

Therefore, $S(3,n,s)$ is C_3-supermagic graph when s even. □

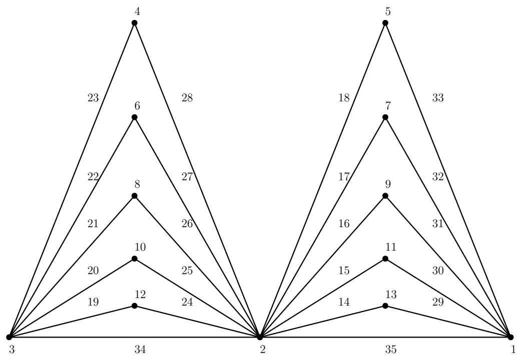

FIGURE 2. C_3-supermagic labelling on $S(3,5,2)$ where $m(\lambda) = 94$.

Theorem 2.3. *Let $n, s \geq 2$ and n even. Then $S(3,n,s)$ is C_3-supermagic.*

Proof. Let's define λ as follows:

$$\lambda(v_i) = i, i = 1, 2, \ldots, s+1,$$
$$\lambda(v_{ij}) = s + 1 + j + (i-1)n, i = 1, 2, \ldots, s, j = 1, 2, \ldots, n,$$
$$\lambda(e_i) = \left(\frac{1}{2} - i\right)n + 2s + 2ns + 2 - i, i = 1, 2, \ldots, s,$$
$$\lambda(e^1_{ij}) = \begin{cases} (1-i)\,n - \frac{1}{2}j + 2s + 2ns + \frac{5}{2} - i, & i = 1,2,3,\ldots,s, j = 1,3,5,...,n-1, \\ \left(\frac{1}{2} - i\right)n - \frac{1}{2}j + 2s + 2ns + 2 - i, & i = 1,2,3,\ldots,s, j = 2,4,6,...,n, \end{cases}$$
$$\lambda(e^2_{ij}) = \begin{cases} 2s - \left(\frac{1}{2} - i\right)n - \frac{1}{2}j + 2ns + \frac{3}{2}, & i = 1,2,3,\ldots,s, j = 1,3,5,...,n-1, \\ in - \frac{1}{2}j + 2s + 2ns + 2, & i = 1,2,3,\ldots,s, j = 2,4,6,...,n. \end{cases}$$

Since we have

$$\sum_{v\in V'}\lambda(v) = v_i + v_{i+1} + v_{ij} = j - (1-i)\,n + s + 2 + 2i$$

and

$$\sum_{e\in E'}\lambda(e)=e_i+e_{ij}^1+e_{ij}^2=(1-i)\,n-j+6s+6ns+6-2i,$$

the supermagic sum $m\left(\lambda\right)$ is

$$m(\lambda)=\sum_{v\in V'}\lambda(v)+\sum_{e\in E'}\lambda(e)=7s+6ns+8.$$

Therefore, $S(3,n,s)$ is C_3-supermagic graph when n even. $\square$

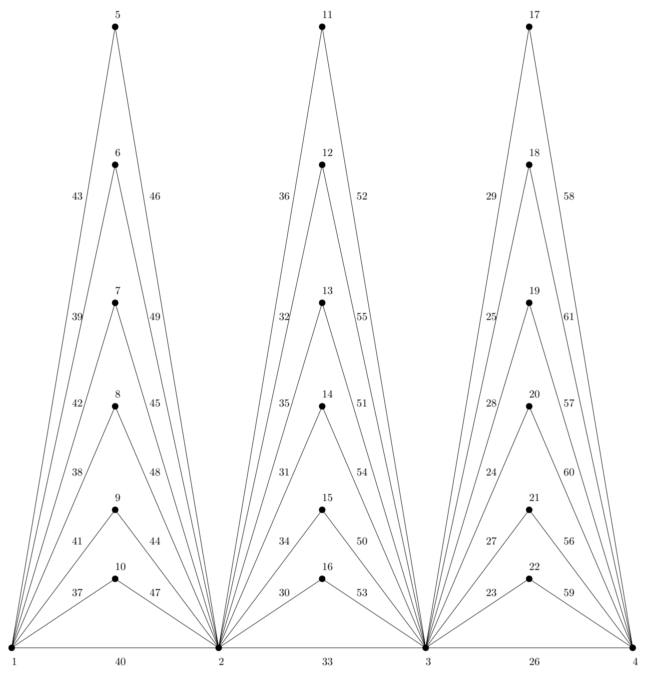

FIGURE 3. C_3-supermagic labelling on $S\left(3,6,3\right)$ where $m\left(\lambda\right)=137$.

Theorem 2.4. *Let $n,s\geq 2,n$ and s odd. Then $S(3,n,s)$ is C_3-supermagic.*

Proof. Let's define λ as follows:

$$\lambda\left(v_i\right)=\begin{cases}\frac{i+1}{2}, & i \text{ odd and } i=1,2,\ldots,s+1,\\ \frac{s+i+1}{2}, & i \text{ even and } i=1,2,\ldots,s+1,\end{cases}$$
$$\lambda\left(v_{ij}\right)=s+j+n(i-1)+1, i=1,2,\ldots,s, j=1,2,\ldots,n,$$
$$\lambda\left(e_i\right)=3ns+2s-i+2, i=1,2,\ldots,s,$$
$$\lambda\left(e_{ij}^1\right)=\begin{cases}\frac{3ns}{2}+s+j+n(i-1)+\frac{1}{2}, & i=1,2,...,\frac{s-1}{2}, j=1,2,...,n,\\ 2ns+s-\frac{n}{2}+j+\frac{1}{2}, & i=\frac{s+1}{2}, j=1,2,...,\frac{n+1}{2},\\ ns+s+1+j-\frac{n+1}{2}, & i=\frac{s+1}{2}, j=\frac{n+1}{2}+1,...,n,\\ \frac{ns}{2}+s+j+n(i-1)+\frac{1}{2}, & i=\frac{s+3}{2},...,s, j=1,2,...,n,\end{cases}$$
$$\lambda\left(e_{ij}^2\right)=\begin{cases}3ns+s-2j-2n(i-1)+3, & i=1,2,...,\frac{s-1}{2}, j=1,2,...,n,\\ 2ns+n+s-2j+3, & i=\frac{s+1}{2}, j=1,2,...,\frac{n+1}{2},\\ 3ns+n+s-2j+3, & i=\frac{s+1}{2}, j=\frac{n+1}{2}+1,...,n,\\ 4ns+s-2j-2n(i-1)+3, & i=\frac{s+3}{2},...,s, j=1,2,...,n.\end{cases}$$

Since we have

$$\begin{aligned}\sum_{v\in V'}\lambda(v) &= v_i+v_{i+1}+v_{ij}\\ &=\begin{cases}j-(1-i)\,n+\frac{3}{2}s+\frac{5}{2}+i, & i=1,2,...,\frac{s-1}{2},\\ j-\frac{1}{2}n+2s+\frac{1}{2}ns+3, & i=\frac{s+1}{2} \text{ and } j=\frac{n+1}{2}+1,...,n,\\ j-\frac{1}{2}n+2s+\frac{1}{2}ns+3, & i=\frac{s+1}{2} \text{ and } j=\frac{n+1}{2}+1,...,n,\\ j-(1-i)\,n+\frac{3}{2}s+\frac{5}{2}+i, & i=\frac{s+3}{2},...,s,\end{cases}\end{aligned}$$

and

$$\begin{aligned}\sum_{e\in E'}\lambda(e) &= e_i+e_{ij}^1+e_{ij}^2\\ &=\begin{cases}(1-i)\,n-j+4s+\frac{15}{2}ns+\frac{11}{2}-i, & i=1,2,...,\frac{s-1}{2},\\ \frac{1}{2}n-j+\frac{7}{2}s+7ns+5, & i=\frac{s+1}{2} \text{ and } j=\frac{n+1}{2}+1,...,n,\\ \frac{1}{2}n-j+\frac{7}{2}s+7ns+5, & i=\frac{s+1}{2} \text{ and } j=\frac{n+1}{2}+1,...,n,\\ (1-i)\,n-j+4s+\frac{15}{2}ns+\frac{11}{2}-i, & i=\frac{s+3}{2},...,s,\end{cases}\end{aligned}$$

the supermagic sum $m\left(\lambda\right)$ is

$$m(\lambda)=\sum_{v\in V'}\lambda(v)+\sum_{e\in E'}\lambda(e)=\frac{11}{2}s+\frac{15}{2}ns+8.$$

Therefore, $S(3,n,s)$ is C_3-supermagic graph when n and s odd. □

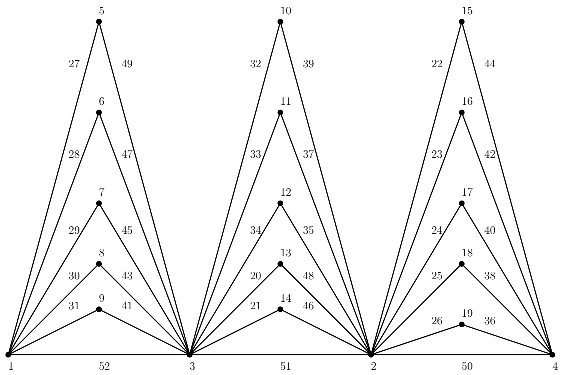

FIGURE 4. C_3-supermagic labelling on $S\left(3,5,3\right)$ where $m\left(\lambda\right)=137$.

As a result of above theorems, we have the following.

Corollary 2.5. *$S(3,n,s)$ is a C_3-supermagic graph.*

2.6. C_m**-supermagicness of** $S\left(m,n,s\right)$**.** We start this subsection with a theorem that we need in the sequel.

For a given graph G, $S(G)$ denote the graph obtained by subdividing some edges of G. The next theorem shows that if G is $H-$supermagic, then $S\left(G\right)$ is $S\left(H\right)-$supermagic.

Theorem 2.7 ([10])**.** *Let G be a $H-$supermagic graph and let $H_i, i=1,2,...,t$, be all subgraphs of G isomorphic to H. If $S(H_i), i=1,2,...,t$, are all subgraphs of $S(G)$ isomorphic to $S(H)$ then the graph $S(G)$ is a $S(H)-$supermagic graph.*

The sketch of the author's proof as follows:
Let $S(G)$ be the subdivision of the G graph obtained by adding new $v_1, v_2, \ldots, v_p$ vertices to the G. Let r be the number of new vertices added to the subgraphs $S(H_i)$. If λ is $H-$supermagic labelling of G, then g defined below is a $S(H)-$supermagic labelling of $S(G)$.

$$g\left(v\right)=\begin{cases}\lambda\left(v\right), & v\in V\left(G\right)\\ \left|V\left(G\right)\right|+j, & v=v_j, j=1,2,\ldots,p\end{cases}$$

$$g\left(uw\right)=\begin{cases}\lambda\left(uv\right)+p, & u\in V\left(G\right)\\ \left|V\left(G\right)\right|+\left|E\left(G\right)\right|+2p+1-j, & u=v_j, j=1,2,\ldots,p\end{cases}$$

In this case the magic sum is

$$\mu\left(g\right)=\mu\left(\lambda\right)+\left|E\left(H\right)\right|p+\left(2\left|V\left(G\right)\right|+\left|E\left(G\right)\right|+2p+1\right)r.$$

Now, we define $m-$polygonal-book-snake graphs with n leaves, s parts where $m\geq 4$ and show that they admit a C_m-supermagic labelling by using above theorem.

The graph with $\left(m-2\right)ns+s+1$ vertex and $\left(m-1\right)ns+s$ edges, obtained by joining s copies of the $m-$polygonal book graph with $n-$leaves $B(m,n)$ along the common vertices, is called $m-$polygonal-book-snake graph with the $n-$leaves and $s-$parts, and denoted by $S\left(m,n,s\right)$ (see Figure 5).

$$V=\{v_i : i=1,2,\ldots,s+1\}\cup\{v_{ij}^k : i=1,2,\ldots,s, j=1,2,\ldots,n, k=1,2,\ldots,m-2\}$$
$$E=\{e_i : i=1,2,\ldots,s\}\cup\{e_{ij}^k : i=1,2,\ldots,s, j=1,2,\ldots,n, k=1,2,\ldots,m-1\}$$

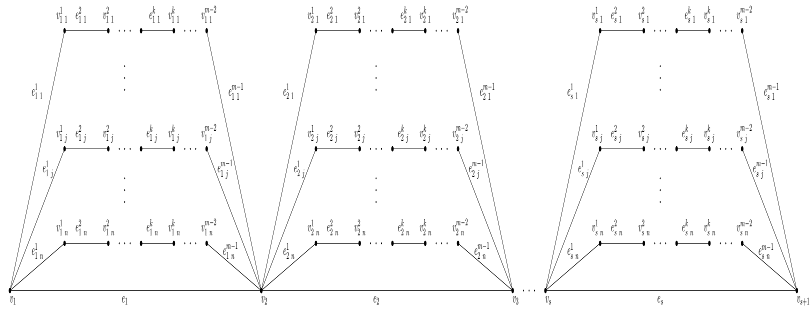

FIGURE 5. $S\left(m,n,s\right)$ graph

$S\left(m,n,s\right)$ can be taught as a subdivision of $S\left(3,n,s\right)$ since it can be obtained by adding new $r=\left(m-3\right)$ vertices between v_{ij} and v_{i+1} for all i and j, totally $p=\left(m-3\right)ns$ vertices. Therefore, by Theorem 2.7, we have the following.

Corollary 2.8. *Let $n,s\geq 2$ and $m\geq 4$. Then $S(m,n,s)$ is C_m-supermagic.*

Proof. By combining the theorems in subsection 2.1 with supermagicness of subdivided graphs, according to the values of n and s, C_m-supermagicness of $S\left(m,n,s\right)$ is as follows:

If s is even:

$$\begin{aligned}
g(v_i) &= s+2-i, i=1,2,3,\ldots,s+1,\\
g(v_{ij}^1) &= s+1+i+s(j-1), i=1,2,3,\ldots,s, j=1,2,\ldots,n,\\
g(v_{ij}^k) &= ns+s+k+(j-1)\left(m-3\right)+(i-1)\left(m-3\right)n, \qquad i=1,2,3,\ldots,s,\\
&\qquad\qquad j=1,2,3,\ldots,n,\\
&\qquad\qquad k=2,3,4,\ldots,m-2,\\
g(e_i) &= 3ns+s+i+1+\left(m-3\right)ns, i=1,2,\ldots,s,\\
g(e_{ij}^1) &= \begin{cases} 2ns+\frac{3s}{2}-\frac{sj}{2}-i+2+\left(m-3\right)ns, & i=1,2,3,\ldots,\frac{s}{2}, j=1,2,3,\ldots,n,\\ \frac{3ns}{2}+2s-\frac{sj}{2}-i+2+\left(m-3\right)ns, & i=\frac{s}{2}+1,\ldots,s, j=1,2,3,\ldots,n,\end{cases}\\
g(e_{ij}^2) &= \begin{cases} 2ns+s+s\frac{n-j}{2}+i+1+\left(m-3\right)ns, & i=1,2,3,\ldots,\frac{s}{2}, j=1,2,3,\ldots,n,\\ 3ns+\frac{s}{2}-\frac{sj}{2}+i+1+\left(m-3\right)ns, & i=\frac{s}{2}+1,\ldots,s, j=1,2,3,\ldots,n,\end{cases}
\end{aligned}$$

$$g(e_{ij}^{k}) = 3j-k+m-(3-3i)n+2s-jm+(1-i)mn-3ns+2mns+1, \quad i=1,2,3,\ldots,s, \; j=1,2,3,\ldots,n, \; k=3,4,5,\ldots,m-1,$$

$$\begin{aligned} m(g) &= \mu(\lambda)+|E(H)|p+(2|V(G)|+|E(G)|+2p+1)r \\ &= 3m-\frac{7}{2}s+3ms+\frac{9}{2}ns+2m^2ns-5mns-1. \end{aligned}$$

If n is even:

$$\begin{aligned}
g(v_i) &= i, i=1,2,\ldots,s+1,\\
g(v_{ij}^1) &= s+1+j+(i-1)n, i=1,2,\ldots,s, j=1,2,3,\ldots,n,\\
g(v_{ij}^k) &= ns+s+k+(j-1)(m-3)+(i-1)(m-3)n, \quad i=1,2,3,\ldots,s,\; j=1,2,3,\ldots,n,\; k=2,3,4,\ldots,m-2,\\
g(e_i) &= \left(\tfrac{1}{2}-i\right)n+2s+2ns+2-i+(m-3)ns, i=1,2,\ldots,s,\\
g(e_{ij}^1) &= \begin{cases} (1-i)n-\frac{1}{2}j+2s+2ns+\frac{5}{2}-i+(m-3)ns, & i=1,2,3,\ldots,s,\; j=1,3,5,\ldots,n-1,\\ \left(\frac{1}{2}-i\right)n-\frac{1}{2}j+2s+2ns+2-i+(m-3)ns, & i=1,2,3,\ldots,s,\; j=2,4,6,\ldots,n, \end{cases}\\
g(e_{ij}^2) &= \begin{cases} 2s-\left(\frac{1}{2}-i\right)n-\frac{1}{2}j+2ns+\frac{3}{2}+(m-3)ns, & i=1,2,3,\ldots,s,\; j=1,3,5,\ldots,n-1,\\ in-\frac{1}{2}j+2s+2ns+2+(m-3)ns, & i=1,2,3,\ldots,s,\; j=2,4,6,\ldots,n, \end{cases}\\
g(e_{ij}^k) &= 3j-k+m-(3-3i)n+2s-jm+(1-i)mn-3ns+2mns+1, \quad i=1,2,3,\ldots,s,\; j=1,2,3,\ldots,n,\; k=3,4,5,\ldots,m-1,
\end{aligned}$$

$$\begin{aligned} m(g) &= \mu(\lambda)+|E(H)|p+(2|V(G)|+|E(G)|+2p+1)r \\ &= 3m-2s+3ms+3ns+2m^2ns-5mns-1. \end{aligned}$$

If s odd and n is odd:

$$\begin{aligned}
g(v_i) &= \begin{cases} \frac{i+1}{2}, & i \text{ odd and } i=1,2,\ldots,s+1,\\ \frac{s+i+1}{2}, & i \text{ even and } i=1,2,\ldots,s+1, \end{cases}\\
g(v_{ij}^1) &= s+j+n(i-1)+1, i=1,2,\ldots,s, j=1,2,\ldots,n,\\
g(v_{ij}^k) &= ns+s+k+(j-1)(m-3)+(i-1)(m-3)n, \quad i=1,2,3,\ldots,s,\; j=1,2,3,\ldots,n,\; k=2,3,4,\ldots,m-2,
\end{aligned}$$

$$g(e_i) = 3ns + 2s - i + 2 + (m-3)\,ns, i = 1, 2, \ldots, s,$$

$$g(e_{ij}^1) = \begin{cases} \frac{3ns}{2} + s + j + n(i-1) + \frac{1}{2} + (m-3)\,ns, & i = 1,2,3,...,\frac{s-1}{2}, \\ & j = 1,2,3,...,n, \\ 2ns + s - \frac{n}{2} + j + \frac{1}{2} + (m-3)\,ns, & i = \frac{s+1}{2}, j = 1,2,3,...,\frac{n+1}{2}, \\ ns + s + 1 + j - \frac{n+1}{2} + (m-3)\,ns, & i = \frac{s+1}{2}, j = \frac{n+1}{2}+1,...,n, \\ \frac{ns}{2} + s + j + n(i-1) + \frac{1}{2} + (m-3)\,ns, & i = \frac{s+3}{2},...,s, \\ & j = 1,2,3,...,n, \end{cases}$$

$$g(e_{ij}^2) = \begin{cases} 3ns + s - 2j - 2n(i-1) + 3 + (m-3)\,ns, & i = 1,2,3,...,\frac{s-1}{2}, \\ & j = 1,2,3,...,n, \\ 2ns + n + s - 2j + 3 + (m-3)\,ns, & i = \frac{s+1}{2}, j = 1,2,3,...,\frac{n+1}{2}, \\ 3ns + n + s - 2j + 3 + (m-3)\,ns, & i = \frac{s+1}{2}, j = \frac{n+1}{2}+1,...,n, \\ 4ns + s - 2j - 2n(i-1) + 3 + (m-3)\,ns, & i = \frac{s+3}{2},...,s, \\ & j = 1,2,3,...,n, \end{cases}$$

$$g(e_{ij}^k) = 3j - k + m - (3-3i)n + 2s - jm + (1-i)mn - 3ns + 2mns + 1, \quad i = 1,2,3,\ldots,s, \; j = 1,2,3,\ldots,n, \; k = 3,4,5,\ldots,m-1,$$

$$m(g) = \mu(\lambda) + |E(H)|\,p + (2\,|V(G)| + |E(G)| + 2p + 1)\,r$$
$$= 3m - \frac{7}{2}s + 3ms + \frac{9}{2}ns + 2m^2ns - 5mns - 1.$$

□

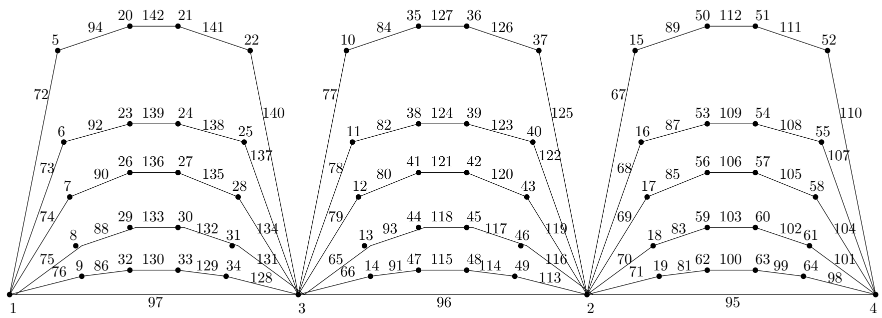

FIGURE 6. C_6-supermagic labelling on $S(6,5,3)$ where $m(\lambda) = 758$.

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